Austrian Academy of Sciences

Acoustic Research Institute – ARI

System Identification in Audio Engineering

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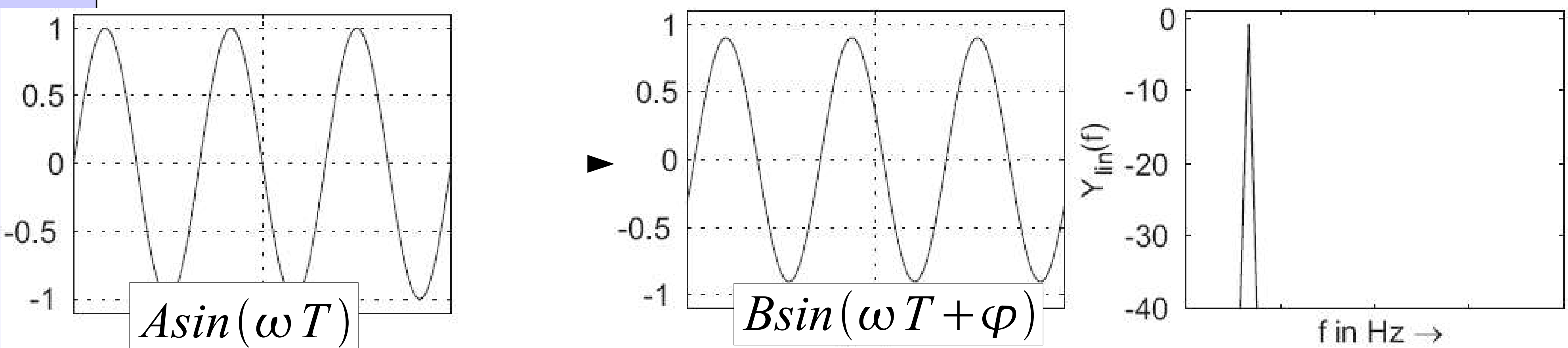
System Identification in Audio Engineering

Devices Under Test (DUT):

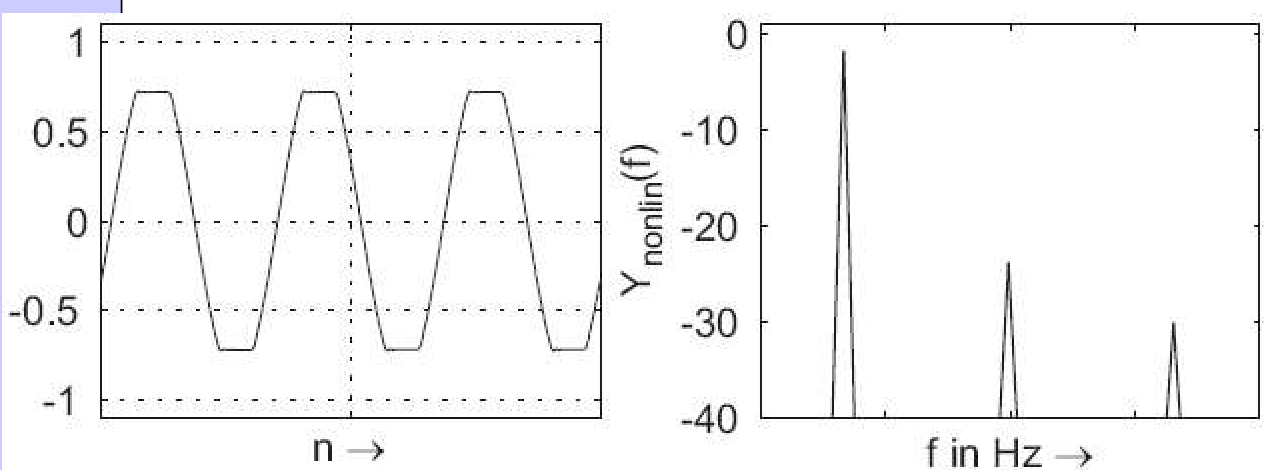
- Acoustical systems
  - Buildings, concert halls, rooms, chambers
  - Acoustical elements: absorber, reflectors
  - *(Sound generators: musical instruments, engines)*
- Electro-acoustic systems
  - Speakers
  - Microphones
- DUT in general: **L**inear, **T**ime **I**nvariant Systems (LTI)

LTI-Systems

- Linearity: $T\{a\cdot x(t)+b\cdot y(t)\}=a\cdot T\{x(t)\}+b\cdot T\{y(t)\}$



– In praxis: weakly nonlinear systems:



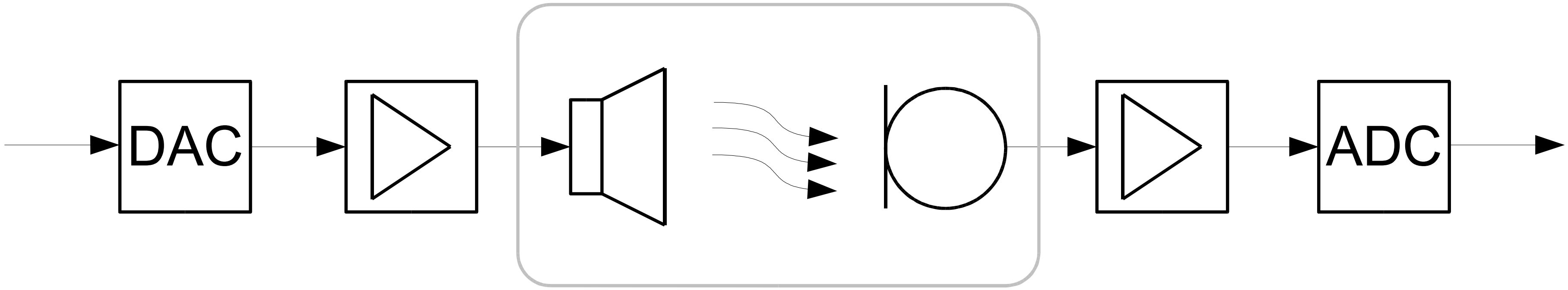
$$A\sin(\omega T) \longrightarrow B\sin(\omega T+\varphi) + \sum_n B_n \sin(n\cdot\omega T+\varphi_n)$$

LTI-Systems

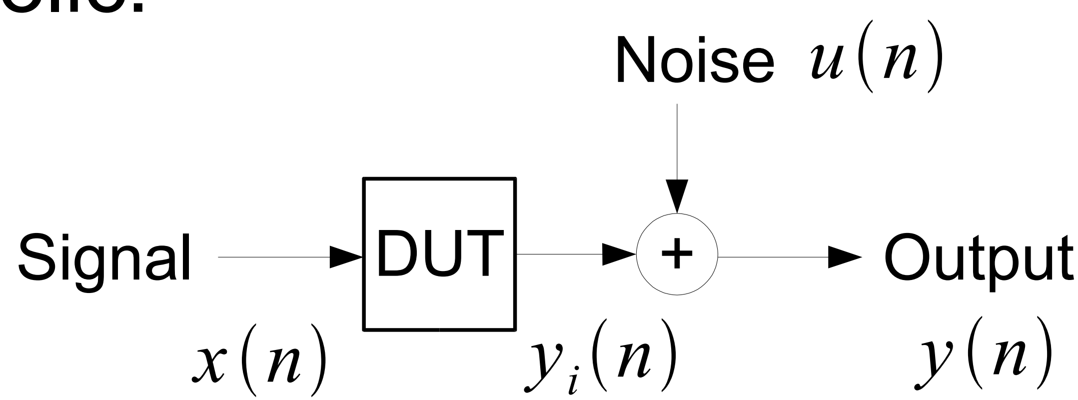
- Time Invariance: $T\{x(t)\}=T\{x(t-T)\}=y(t)$
  - In praxis: a priori knowledge is essential:
    - estimation of time constants
    - repetition after decay of internal energy
  - For nearly time variant systems:
    - short measurement time slot
    - measurement at steady state
- Additional problem: Noise
  - internal noise in measurement equipment
  - internal noise in DUT

Measurement path

- Devices:



- Symbolic:



Parameters

- Transfer Function
  - Spectrum $\underline{H}(f), \underline{H}(k)$
  - Impulse Response $h(t), h(n)$
    - Finite (FIR), Infinite (IIR)
  - Pole-Zero-Diagram
- Signal-To-Noise-Ratio: $SNR = 20 \log_{10}\left(\frac{y_i(n)}{u(n)}\right)$

$E_{signal} \rightarrow max$

→ $A_{RMS} \rightarrow \hat{A}$

→ $\hat{A} / A_{RMS} \rightarrow 1$

→ Minimize the crest factor!

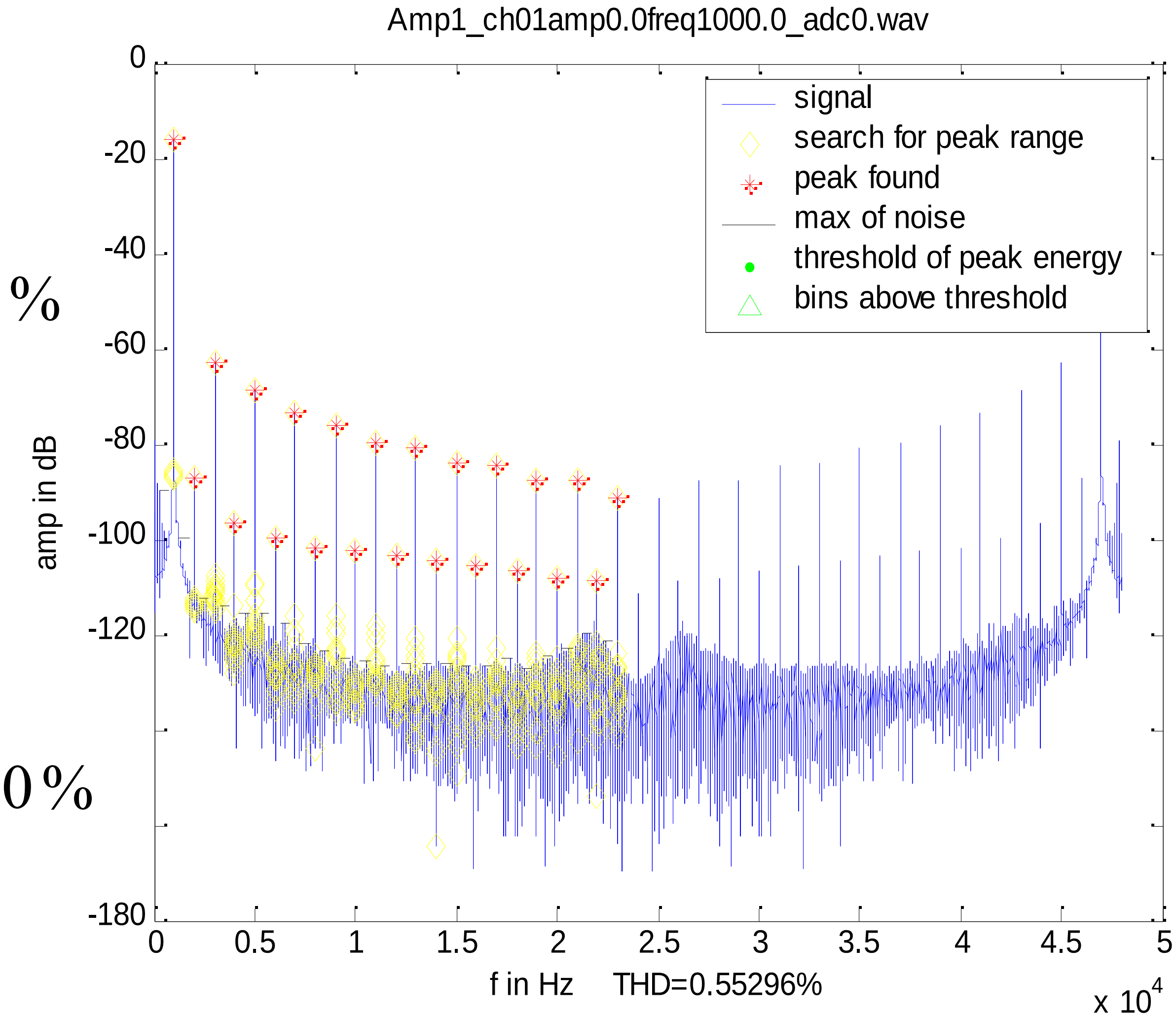
Parameters - Total Harmonic Distortion

- THD:

$$THD = \sqrt{\frac{\sum_{n=2} A_n^2}{A_1^2 + \sum_{n=2} A_n^2}} \cdot 100\,\%$$

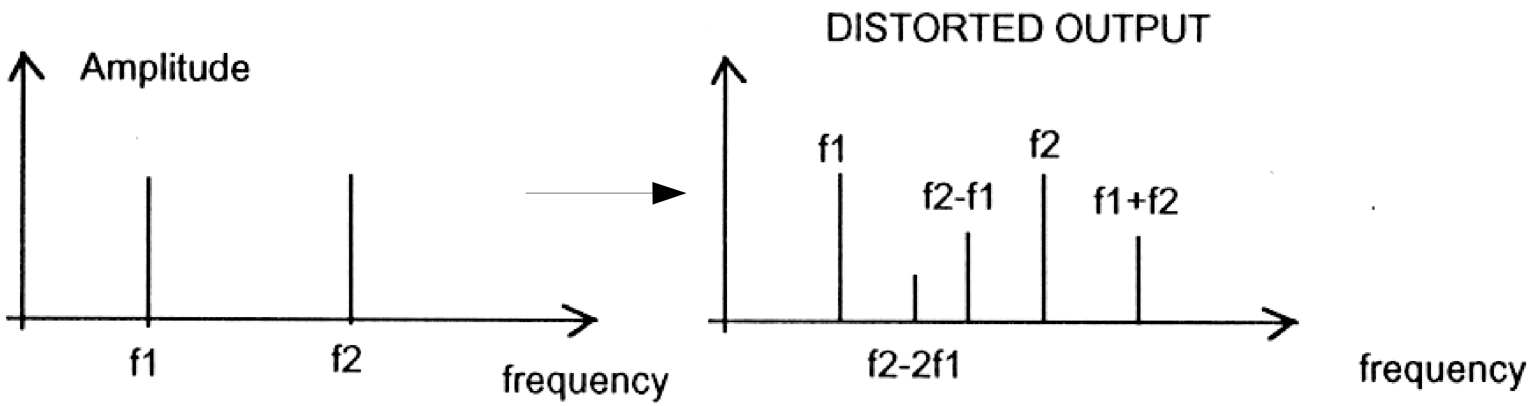
- THD+N:

$$THD + N = \sqrt{\frac{E_{\text{out}} - E_{\text{in}}}{E_{\text{out}}}} \cdot 100\,\%$$



THD+N $\xrightarrow{\text{dB}}$ Signal In Noise And Distortion

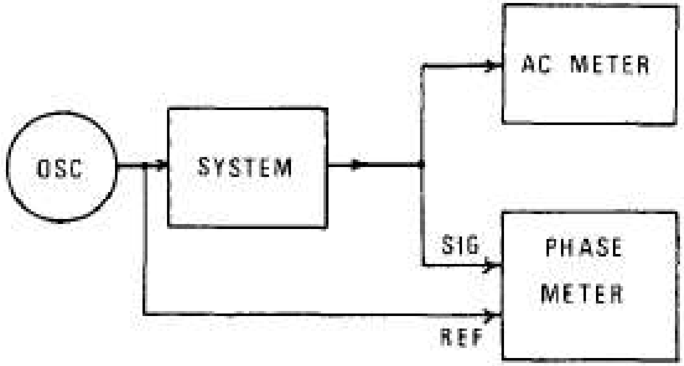
Parameters - Intermodulation Distortions



$$IMD = \left| \frac{A_{f1,f2}}{\sqrt{A_{f1} \cdot A_{f2}}} \right| \cdot 100\,\%$$

Direct Measurement Method

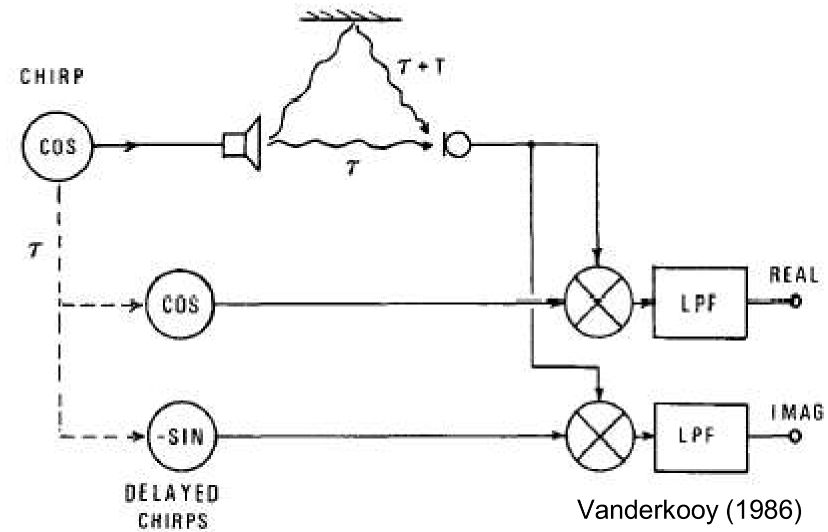
- Direct measurement of amplitude and phase:



Vanderkooy (1986)

- Simple procedure (Stepped Sine)
- Measurement in steady state!!!
  - Long duration for high frequency resolution
- High SNR (crest factor: $\sqrt{2}$)
- Improvement: Time Delay Spectrometry

Time Delay Spectrometry



Vanderkooy (1986)

- Signal: Linear Sweep

$$x(t)=\cos(\omega_0 t)$$

- Response:

$$y(t)=|H(\omega_0)|\cos\left[\omega_0 t+\varphi(\omega_0)\right]$$

$$y(t)=|H(\omega_0)|\left\{\cos(\omega_0 t)\cos\left[\varphi(\omega_0)\right]-\sin(\omega_0 t)\sin\left[\varphi(\omega_0)\right]\right\}$$

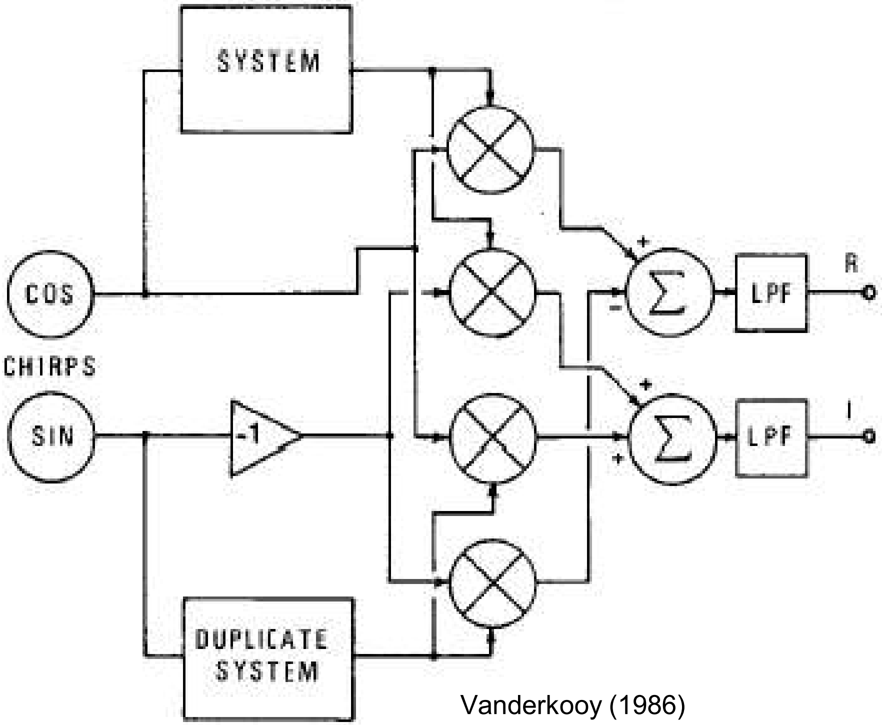
- Demodulation $\frac{1}{2}[1+\cos(2\omega_0 t)]\cdot\cos[\varphi(\omega)]$

  and LP-Filtering:

$$y_R(t)=\frac{1}{2}|H(\omega_0)|\cos\left[\varphi(\omega_0)\right] \qquad y_I(t)=\frac{1}{2}|H(\omega_0)|\sin\left[\varphi(\omega_0)\right]$$

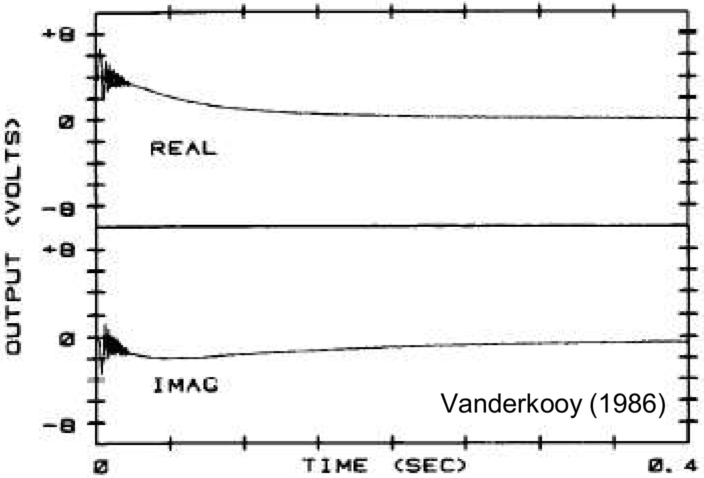
2 pass TDS

- TDS with 2 passes:



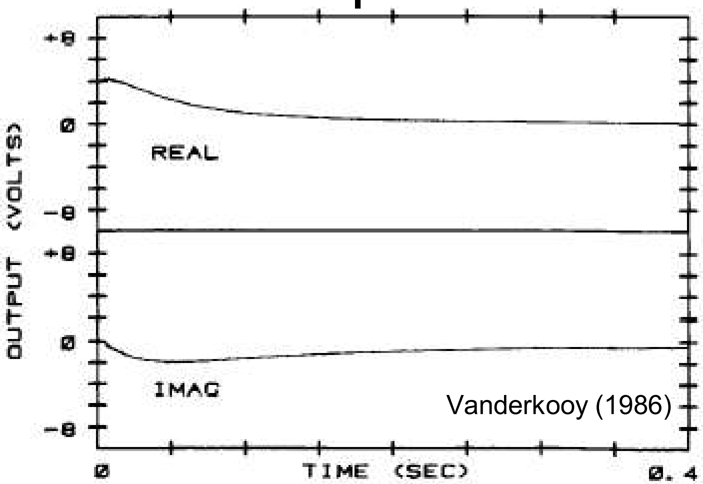
Vanderkooy (1986)

1 pass



Vanderkooy (1986)

2 pass



Vanderkooy (1986)

Impulse Excitation

- Signal: unit pulse with amplitude A
  - Impulse response immediately available
  - Little energy in the excitation signal: $E_{signal} = A^2$
  - High crest factor: $A / A_{RMS} = A \cdot N$ ⟶ low SNR

- Averaging necessary:
  - Periodic Impulse Excitation (PIE)
  - doubling the no. of pulses: +3dB SNR

1- or 2-channel FFT

- Signal: white noise:
  - Amplitude spectrum: DC=0, the rest=1
  - Phase spectrum: random (equal distributed)
  - Signal: random, gaussian distribution
  - Crest factor:
    - Example: fs=48kHz, T=1.5s
      → 72000 samples

Crest Factor	Probability
1	32,00%
2	4,80%
3	0,37%
3,3	0,10%
3,9	0,01%
4	63 ppm
4,4	10 ppm

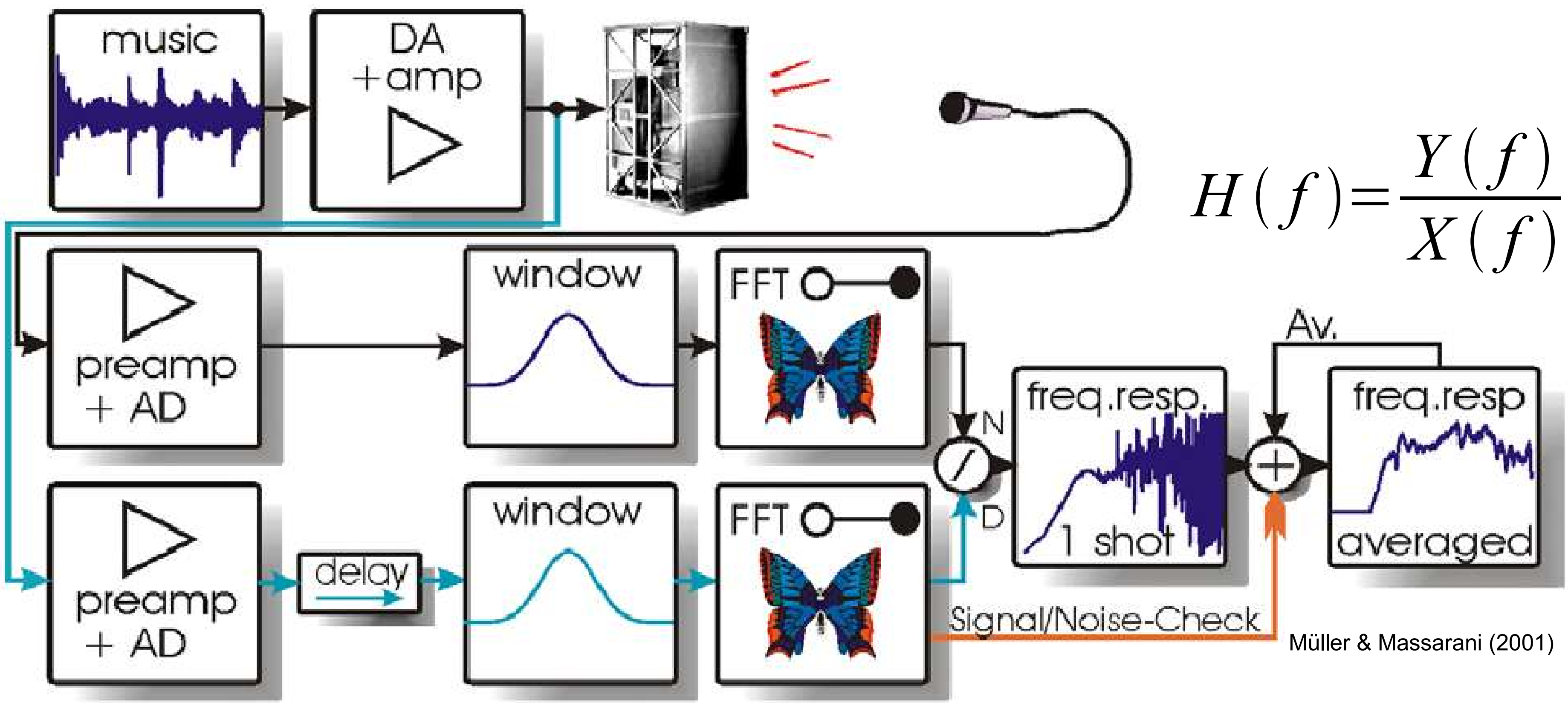
- Averaging necessary
- 1-channel-FFT: for amplitude spectrum only
- 2-channel-FFT: for total transfer function

2-channel FFT

- Identification of amplitude and phase spectrum:

$X(f)$ $Y(f)$

$$H(f)=\frac{Y(f)}{X(f)}$$



Müller & Massarani (2001)

Pseudo Random Sequences

- From the system theory:

$$r_{xy}(n)=h(n)*r_{xx}(n)$$

- White noise – decorrelated signal:

$$r_{xx}(n)=\delta(n)$$

- With a decorrelated signal: $r_{xy}(n)=h(n)$
  - Substitution: white noise → decorrelated signal
  - Wanted:
    - decorrelated signal
    - determinist
    - crest factor of 1

  → binary pseudo random sequences

Maximum Length Sequence (MLS)

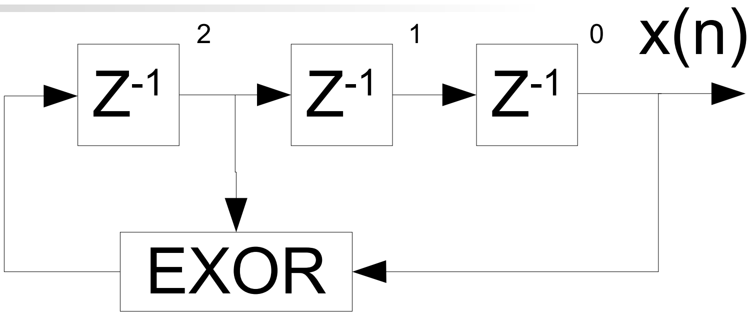
- Generation:
  - N shift registers
  - feedback with EX-OR
- Length of sequence:

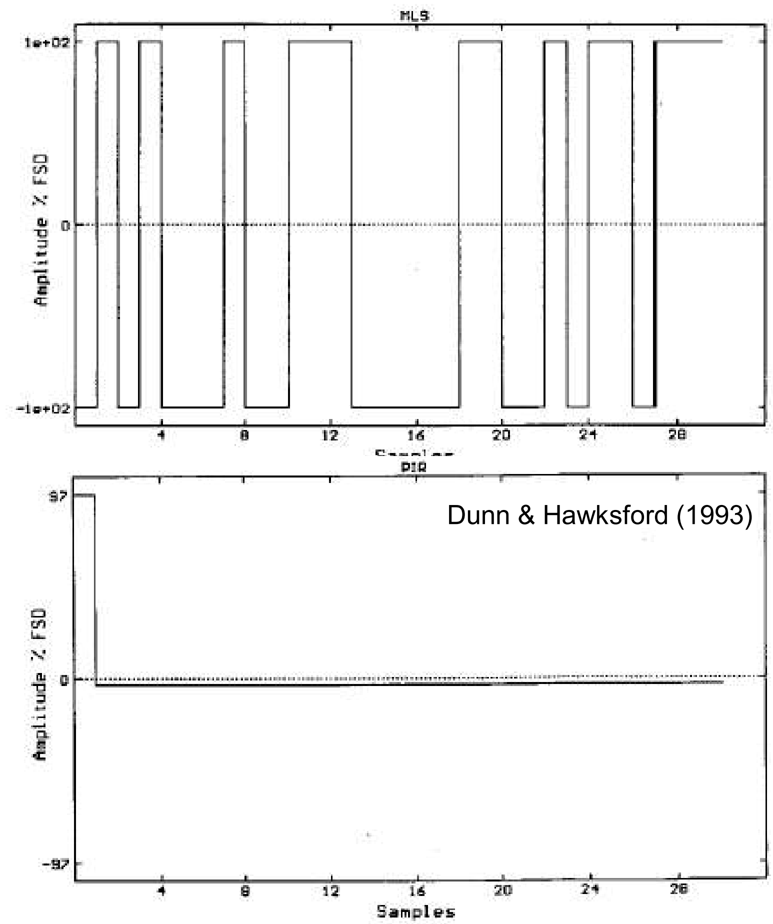
$$L = 2^N - 1$$

- Autocorrelation:

$$r_{xx}(n) = \delta(n) - \frac{1}{L+1}$$

  - Unit pulse with a little offset





MLS

- Calculation of the IR:

$$r_{xx}(n)=\delta(n)-\frac{1}{L+1} \qquad r_{xy}(n)=h(n)*r_{xx}(n)$$

$$r_{xy}(n)=h(n)-\frac{1}{L+1}\sum_{n=0}^{N-1}h(n) \qquad -\frac{1}{L+1}=-\frac{1}{L}+\frac{1}{L(L+1)}$$

$$r_{xy}(n)=h(n)-\frac{1}{L}\sum_{n=0}^{N-1}h(n)+\frac{1}{L(L+1)}\sum_{n=0}^{N-1}h(n)$$

Mittelwert von h(n) = DC

$$\frac{1}{L+1}\cdot DC$$

$$\rightarrow \quad r_{xy}(n)=h(n)-DC\cdot\left[1-\frac{1}{L+1}\right]$$

MLS

- AC-Coupling: $r_{xy}(n) = h(n)$
- DC-Coupling: $r_{xy}(n) \simeq h(n) - DC$
- Calculation of the cross-correlation:
  - frequency domain: FT (not FFT!)
  - direct method:

$$r_{xy}(n) = \frac{1}{L+1} \sum_{i=0}^{L-1} x([i-n] \, \boldsymbol{mod} \, L) \cdot y(i)$$

  - Signal as a matrix:
    - create a circular matrix ***X*** from x(n)
    - Calculation:

$$\boldsymbol{r}_{xy} = \frac{1}{L+1} \boldsymbol{X} \cdot \boldsymbol{y}$$

→ Hadamard Transformation

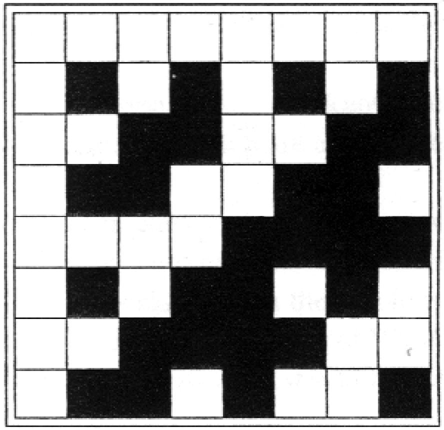
Fast Hadamard Transformation (FHT)

- Algorithmus ähnlich der DFT: $\boldsymbol{X}(k) = \mathscr{F} \cdot \boldsymbol{x}(n)$
- Matrix-Operator: Hadamard-Matrix:

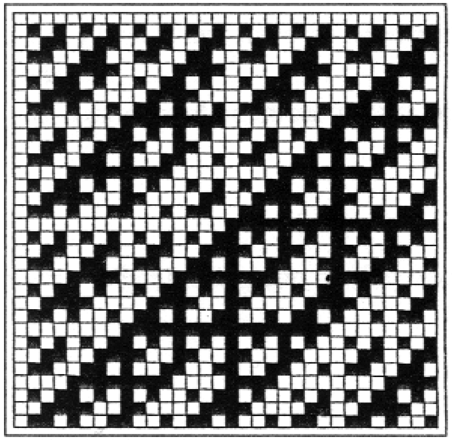
$$\boldsymbol{H}_1 = [1] \qquad \boldsymbol{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\boldsymbol{H}_{2^{n+1}} = \begin{bmatrix} \boldsymbol{H}_{2^n} & \boldsymbol{H}_{2^n} \\ \boldsymbol{H}_{2^n} & -\boldsymbol{H}_{2^n} \end{bmatrix} = \boldsymbol{H}_{2^n} * \boldsymbol{H}_{2^n}$$

$\boldsymbol{H}_8$



$\boldsymbol{H}_{32}$



- Algorithmus:
  - Butterfly, aber kein Bit-reversal
  - nur Additionen/Subtraktionen

MLS

- MLS ist keine Hadamard-Matrix
- Umformung Hadamard-Matrix zur MLS-Matrix:

$$X_{2^n-1} = P_2 S_2 H_{2^n} S_1 P_1$$

$P_{1,} P_2$ ... Permutationsmatrizen

$S_{1,} S_2$ ... Begrenzungsmatrizen (supress)

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{xy} = \frac{1}{L+1} P_2 \left(S_2 \left\{ H_{2^n} \left[S_1 \left(P_1 Y \right) \right] \right\} \right)$$

System Identification with MLS

- Length of MLS:
  - at least the length of the expected IR
- SNR:
  - crest factor: 1 → best SNR we can get!
  - $10\log(L+1)$ higher than PIE
    - fs=48kHz, T=0.7s: → SNR gain: +45dB

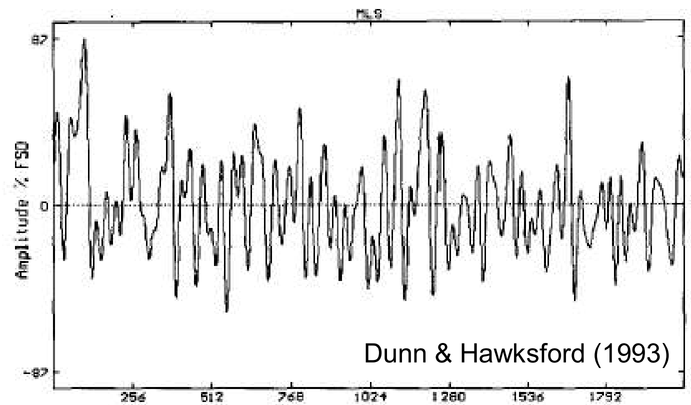
      → doubling PIE length 15 times

      → measurement time with PIE: **9 hours**
- Averaging of time variations
- Very sensitive to nonlinear distortions

Sensitivity of MLS on distortions

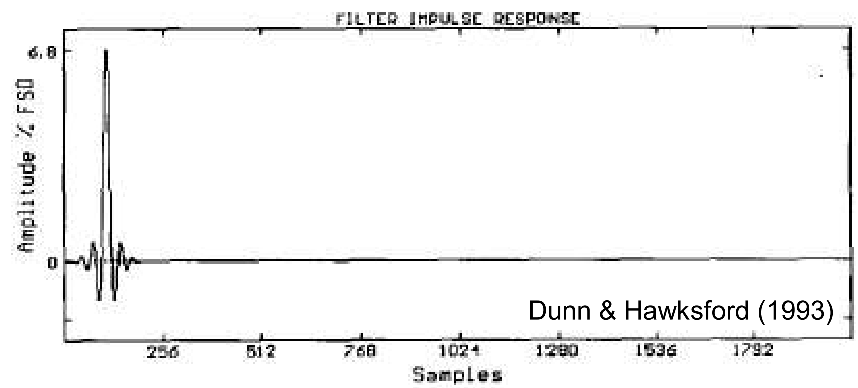
- System:

  LP-Filter, f_c=1kHz



Dunn & Hawksford (1993)

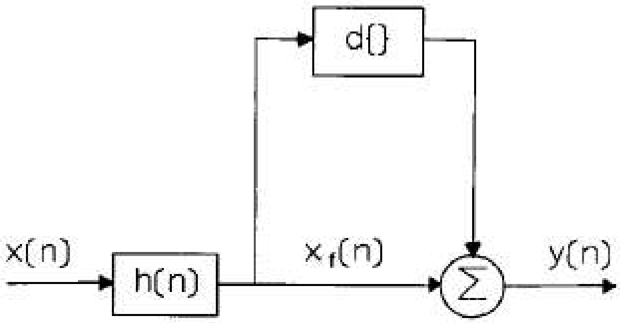
- System output:
- IR of filter:



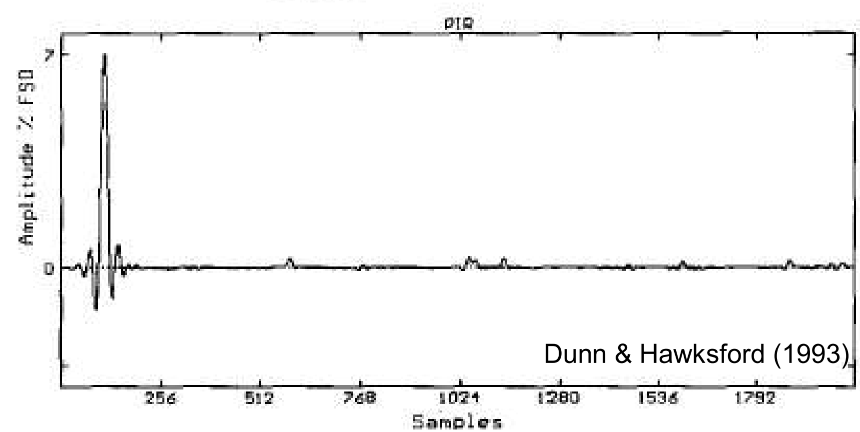
Dunn & Hawksford (1993)

- With distortion:

$$d\left\{x_f(n)\right\} = -10\,dB \cdot \left[x(n)\right]^3$$



Dunn & Hawksford (1993)



Dunn & Hawksford (1993)

Sensitivity of MLS on distortions

- Error signal:

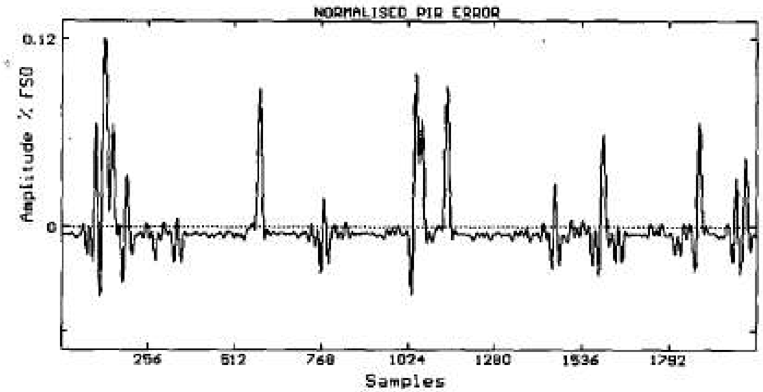
$$e(n)=h_d(n)-h(n)$$

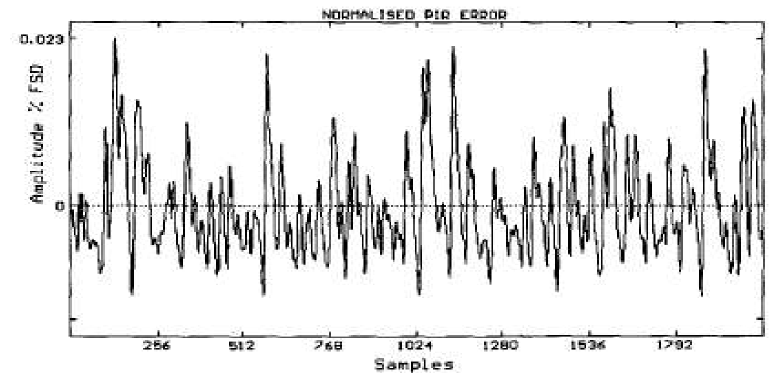
$$h_d(n)=r_{x_f y}(n)+r_{dy}(n)$$

$$h_d(n)=r_{xy}(n)=r_{(x_f+d)y}(n)$$

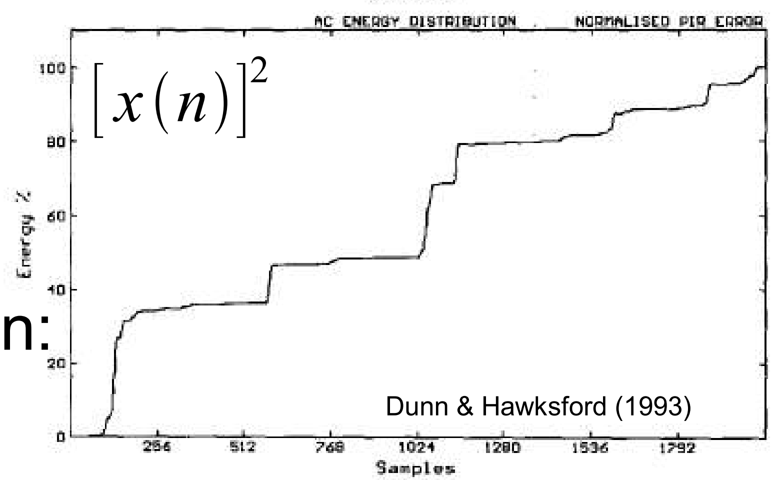
$$e(n)=r_{xd}(n)$$

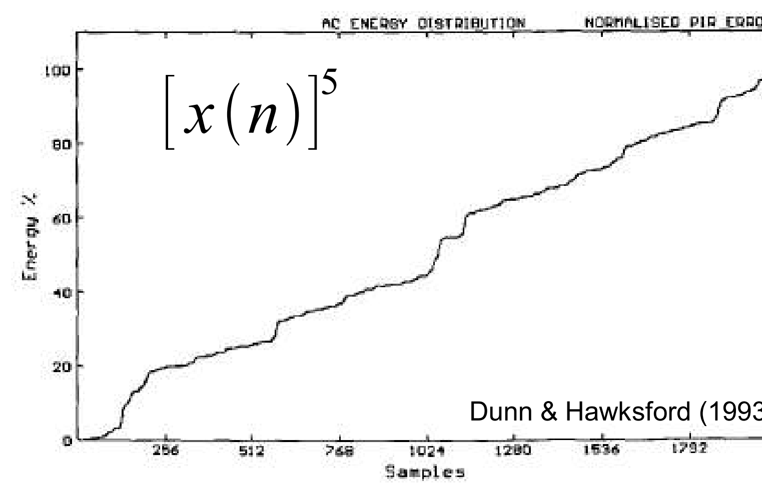
Error:





Energy distribution:





System identification procedure with MLS

- Immunity against noise:

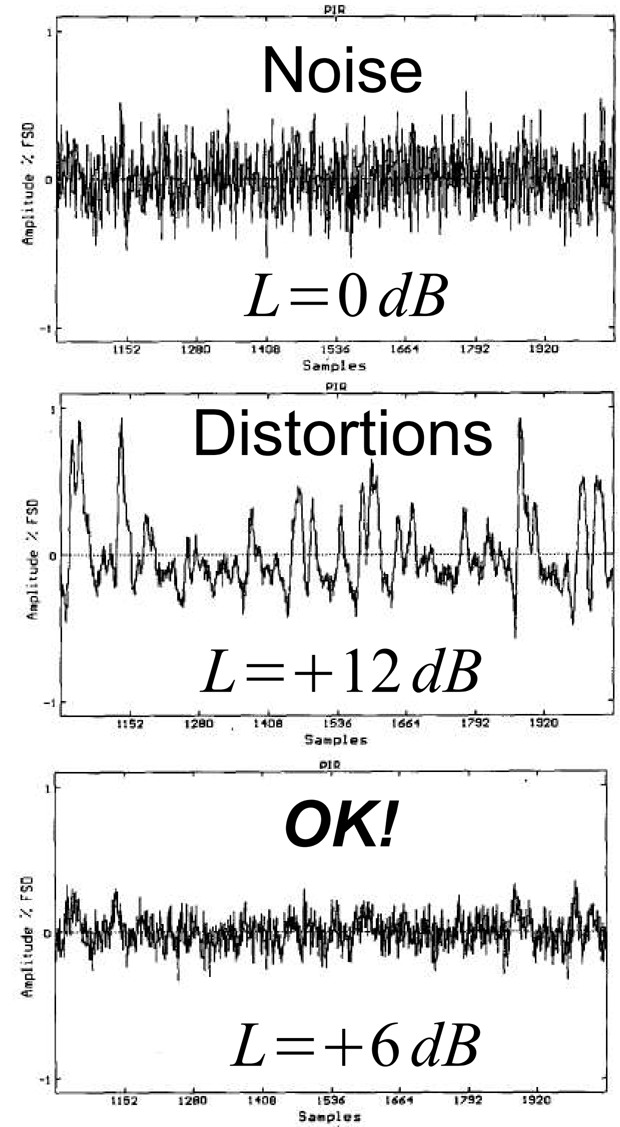
$$\Delta I_n = \Delta A$$

- Immunity against distortions:

$$\Delta I_d = -(r-1) \cdot \Delta A$$

- Depending on system: optimal excitation amplitude

- Increase MLS order instead of length doubling



Dunn & Hawksford (1993)

Inverse Repeated Sequence (IRS)

- Canceling distortions for even orders: $x(n+L) = -x(n)$
- IRS:

$$x(n) = \begin{cases} m(n), & n\,even,\, 0 \le n < 2L \\ -m(n), & n\,odd,\, 0 \le n < 2L \end{cases}$$

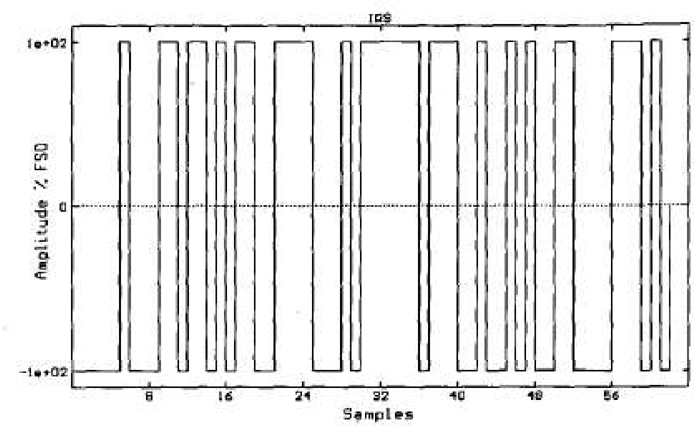
m(n) ... MLS

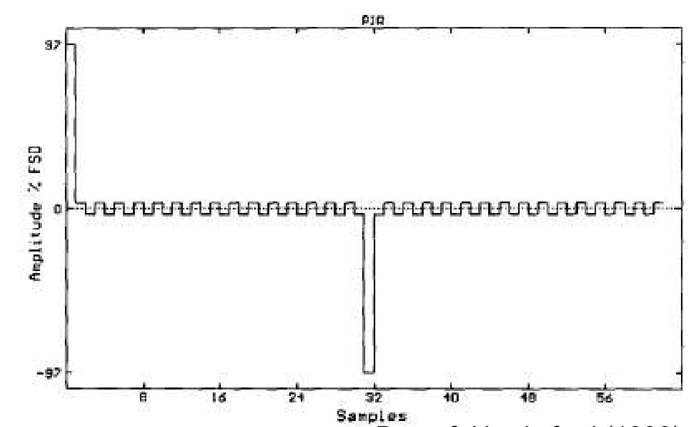
$$r_{xy} = \frac{1}{2(L+1)} \sum_{k=0}^{2} L-1\, x(n)\, x(n+k)$$

$$= \begin{cases} r_{my}(n), & n\,even \\ -r_{my}(n), & n\,odd \end{cases}$$

$$= \delta(n) - \frac{(-1)^n}{L+1} - \delta(n-L) \qquad 0 \le n < 2L$$

Dunn & Hawksford (1993)





Dunn & Hawksford (1993)

Comparison: PIE, MLS, IRS

- Distortion Immunity:

Filter: LP
f = 1kHz
Distortion: -20dB
Length: 2047 samples

Distortion Order	PIE Distortion Immunity (dB)	MLS Distortion Immunity (dB)	IRS Distortion Immunity (dB)
2	54.7	29.4	>262
3	77.2	35.4	36.6
4	99.7	35.9	>265
5	123	38.4	41.4
6	146	39.7	>267
7	169	41.4	46.2

Dunn & Hawksford (1993)

- Noise Immunity (normalized to distortion immunity):

Distortion Order	Distortion Immunity (dB)	Relative MLS Excitation Amplitude (dB)	MLS Noise Immunity Advantage (dB)
2	54.7	−25.3	7.8
3	77.2	−20.9	12.2
4	99.7	−21.3	11.8
5	123	−21.2	11.9
6	146	−21.3	11.8
7	169	−21.3	11.8

Dunn & Hawksford (1993)

Improving Distortion Immunity

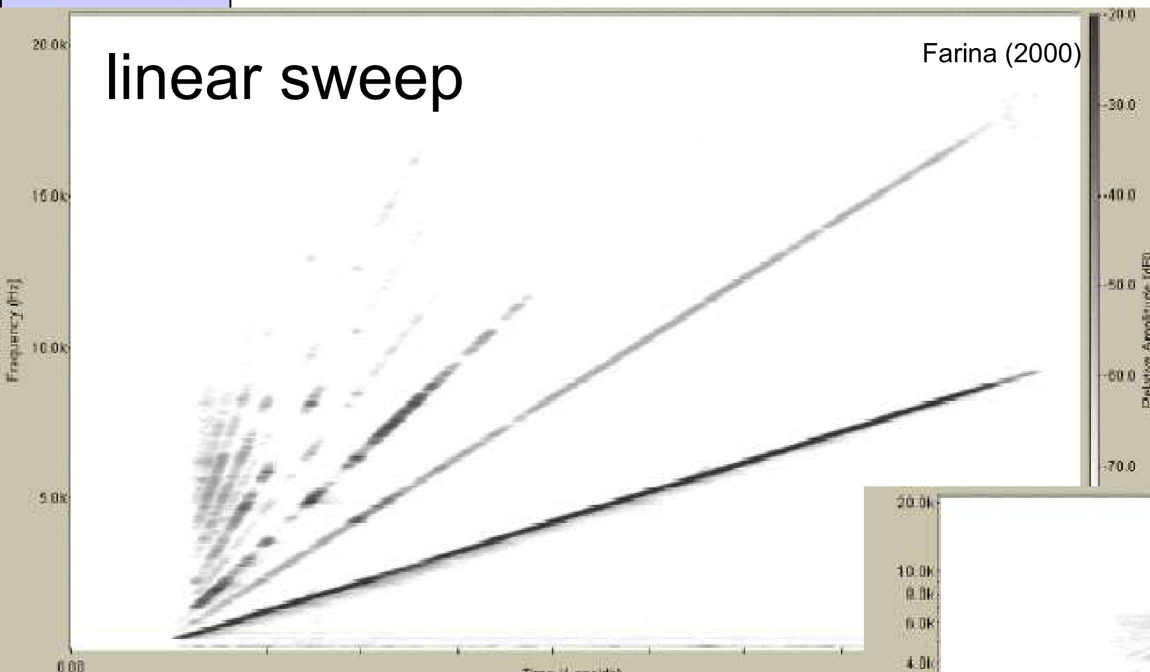
- Problems of MLS:
  - Sensitivity to distortions
- What we want:
  - Identification of the linear part
  - All harmonics separated
- Solution:
  - Exponential sweep: $x(t) = \sin\left[A\left(e^{t/\tau} - 1\right)\right]$

$$A = \frac{T\omega_1}{\ln\left(\omega_2/\omega_1\right)}$$

$$\tau = \frac{T}{\ln\left(\omega_2/\omega_1\right)}$$

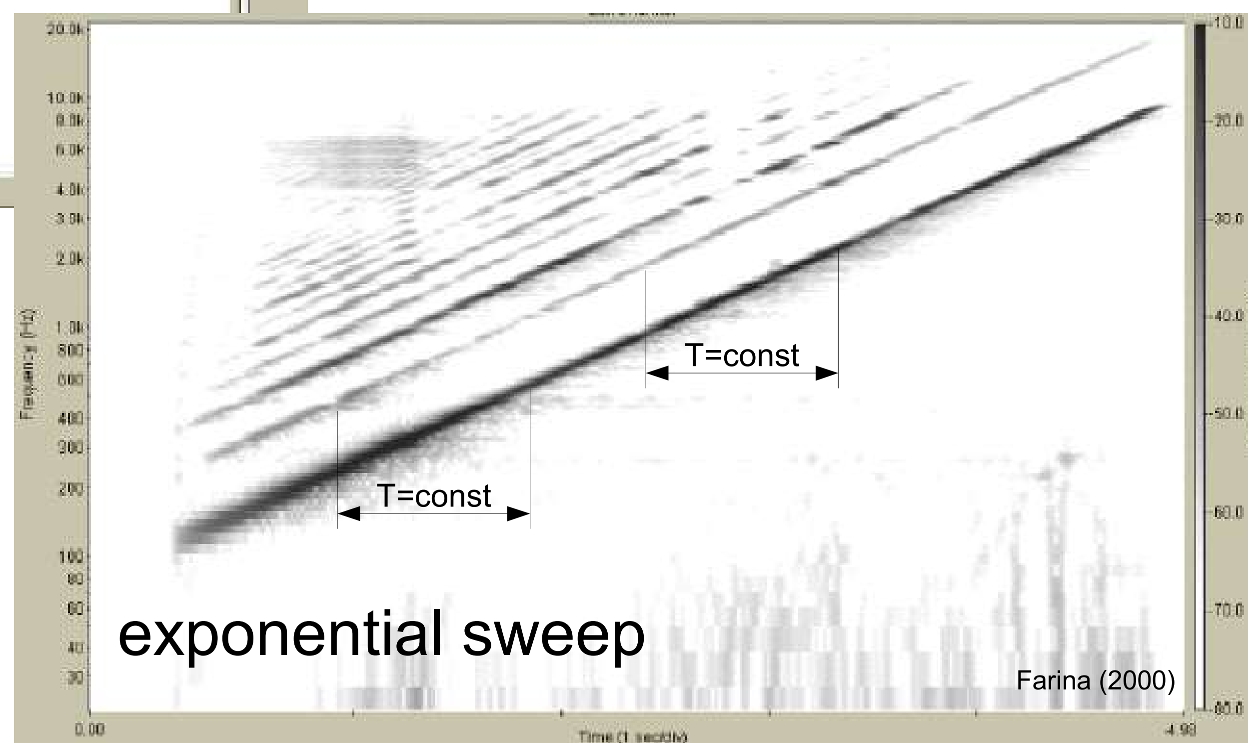
with:
T ... sweep length
ω_1 ... start frequency
ω_2 ... end frequency

Exponential Sweep



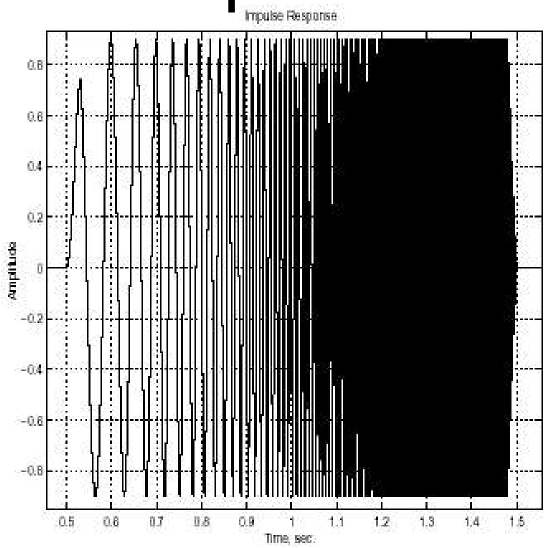
Weakly nonlinear system as example

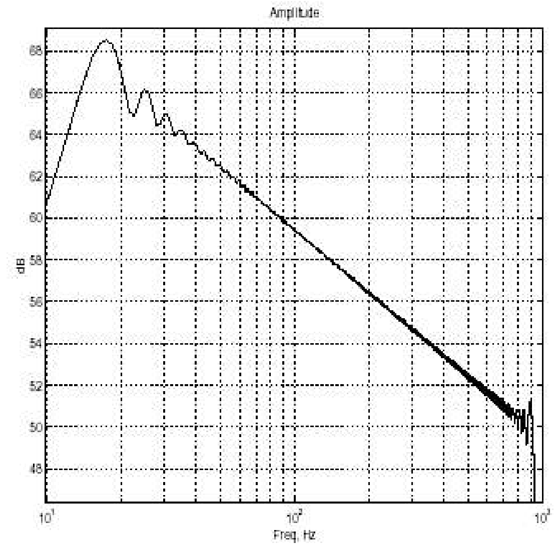
Constant distance between all harmonics



Sweep Response To Impulse Response

– Sweep:



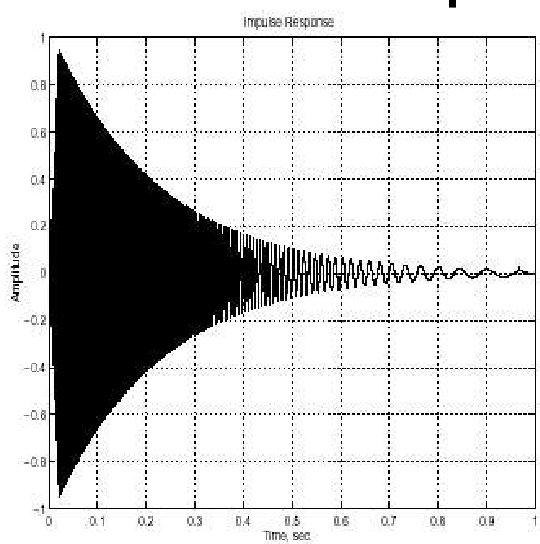


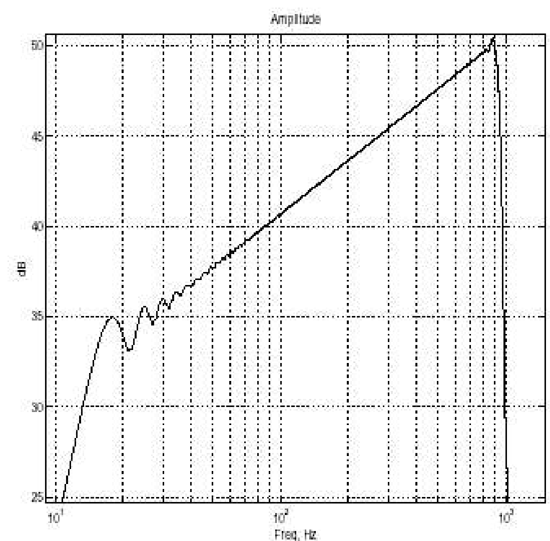
$$Y(f) = X(f) \cdot H(f)$$

$$\downarrow$$

$$H(f) = Y(f) \cdot X^{-1}(f)$$

– Inverse sweep:



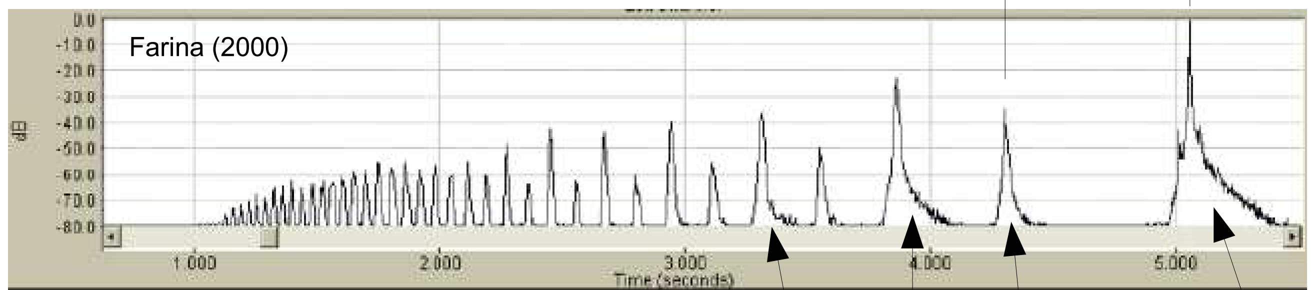


$$X^{-1}(f) = \frac{\mathscr{F}\{x(-t)\}}{X(f)^2}$$

Separating Harmonics

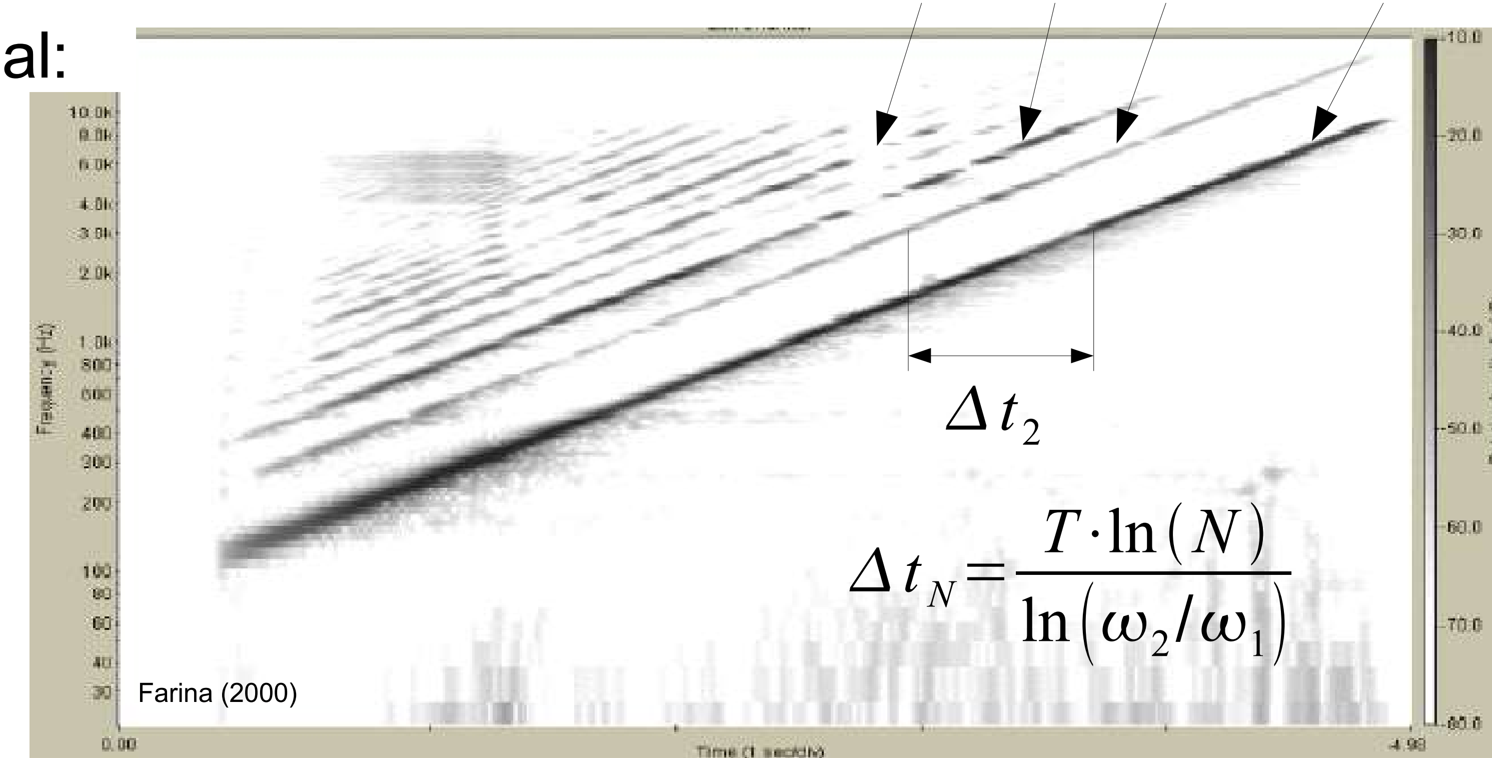
Δt_2

Impulse Response:



IR of harmonics: 5th 3rd 2nd linear

Output signal:



Δt_2

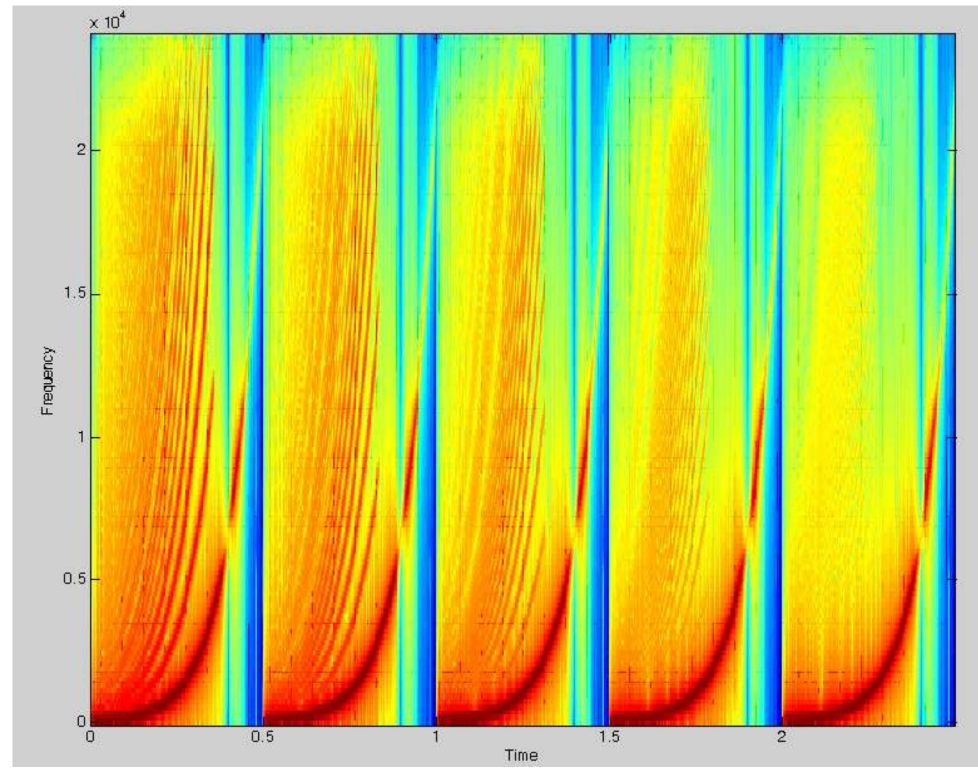
$$\Delta t_N = \frac{T \cdot \ln(N)}{\ln(\omega_2/\omega_1)}$$

Simultaneous Measurement of THD and IR

Amplitude spectra of separated harmonics:

THD @ 1kHz

Measurement of multiple systems

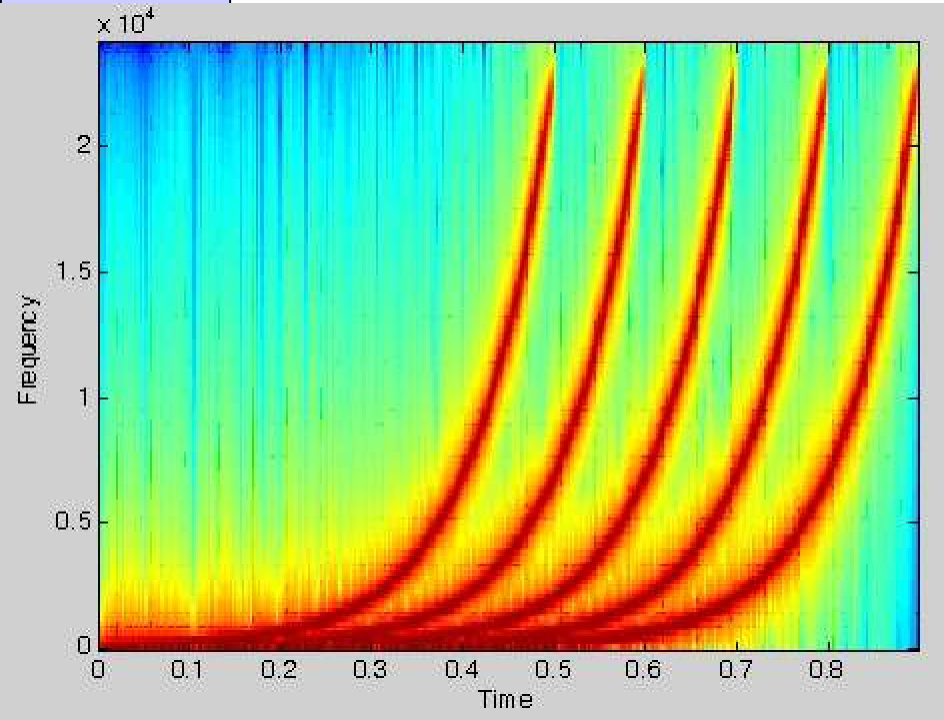


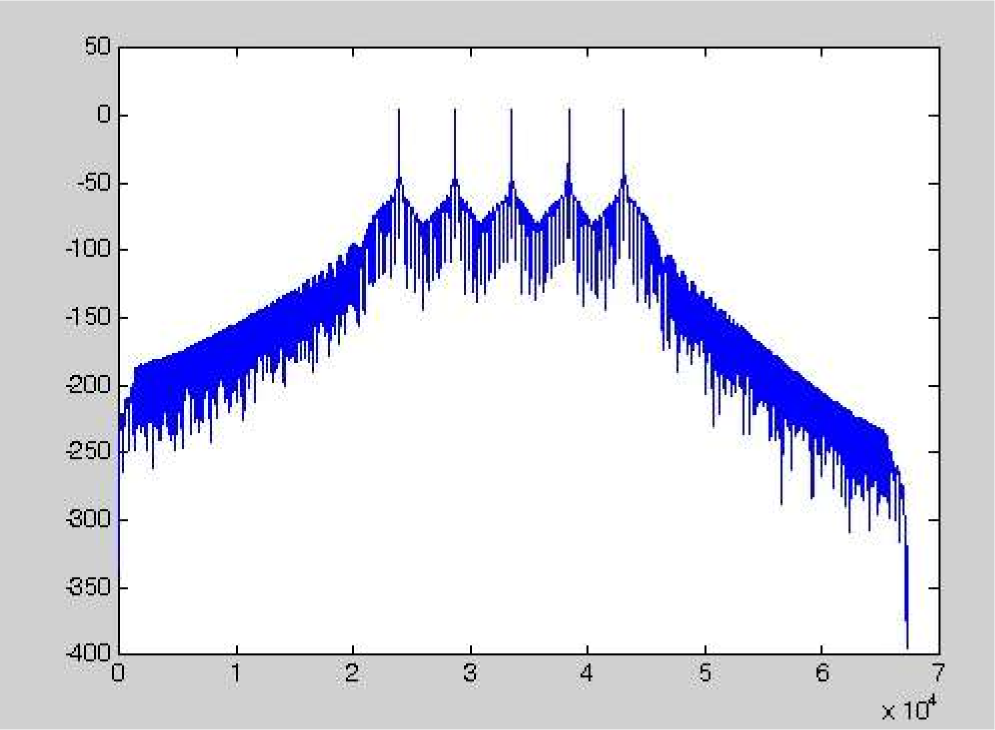
1155 systems á 1.5 sec.

Exp Sweep:	29'
MLS:	(13') 52'
PIE:	38.5 days

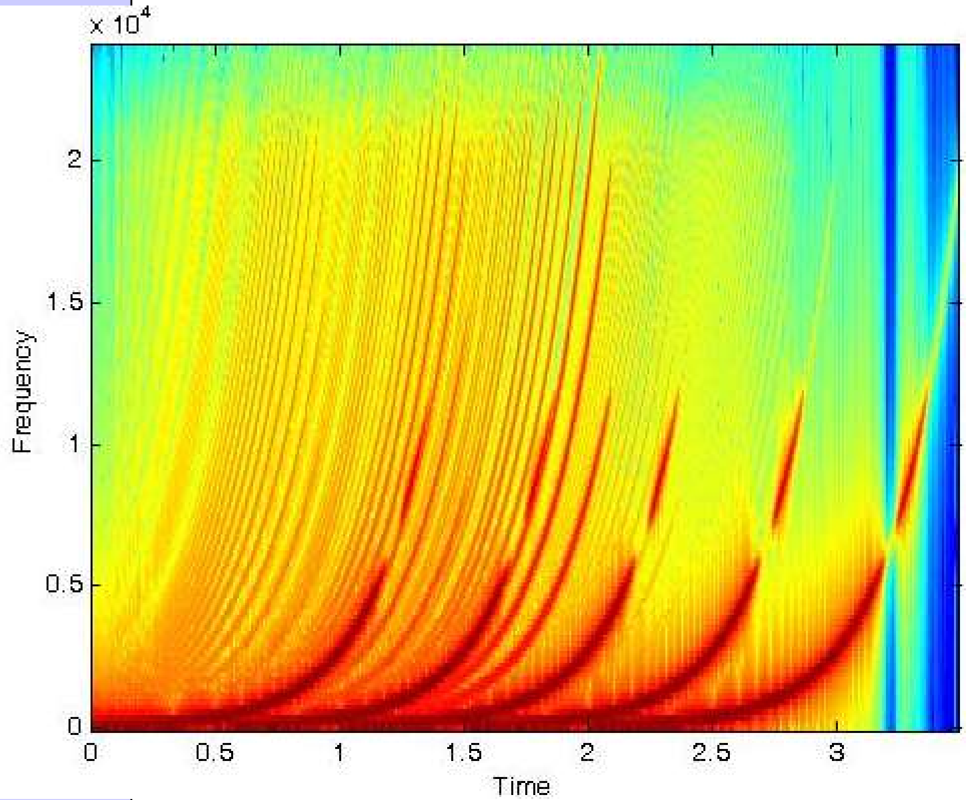
AAS-ARI

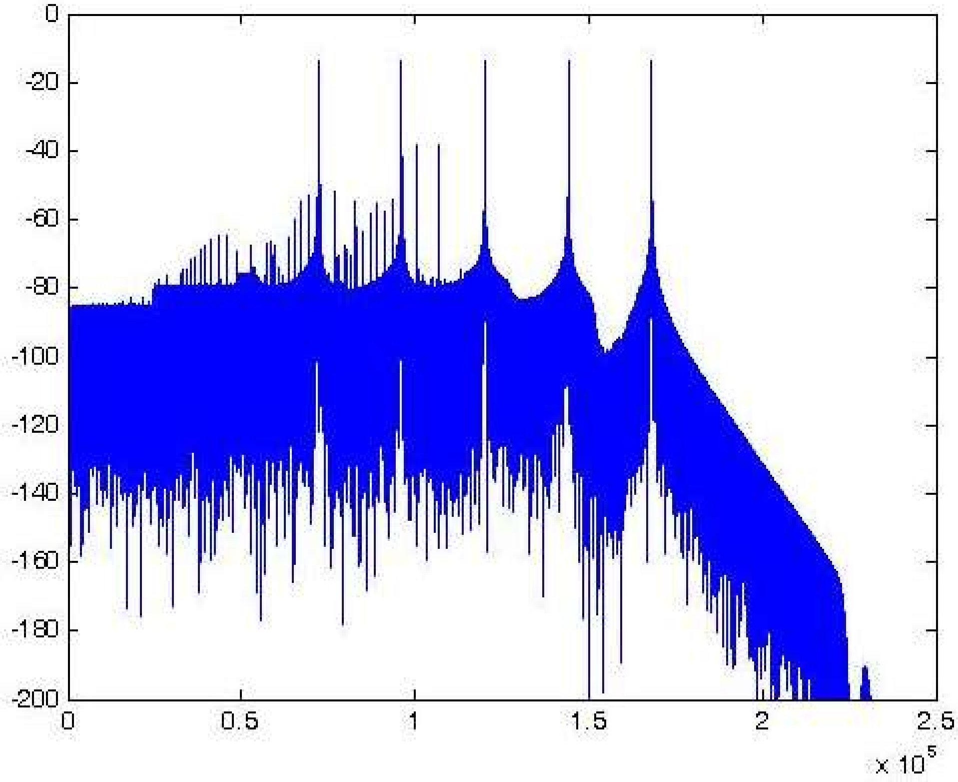
Measurement of multiple systems





Measurement of multiple systems

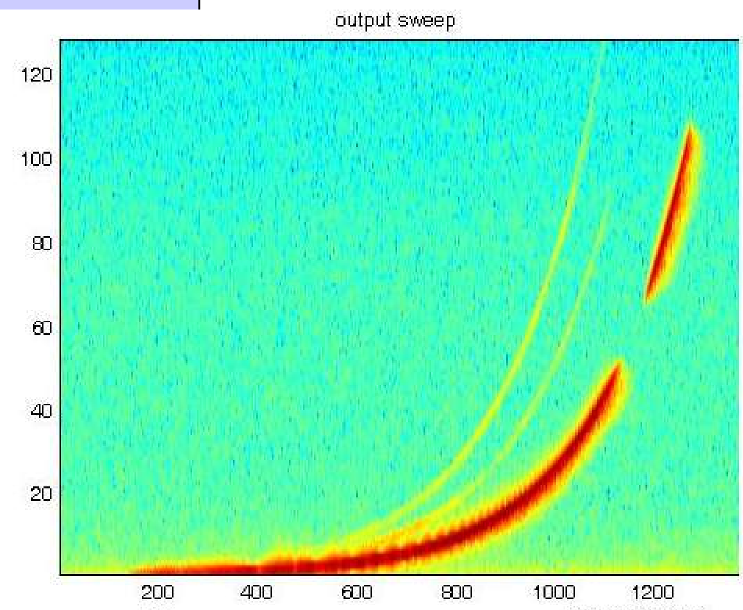


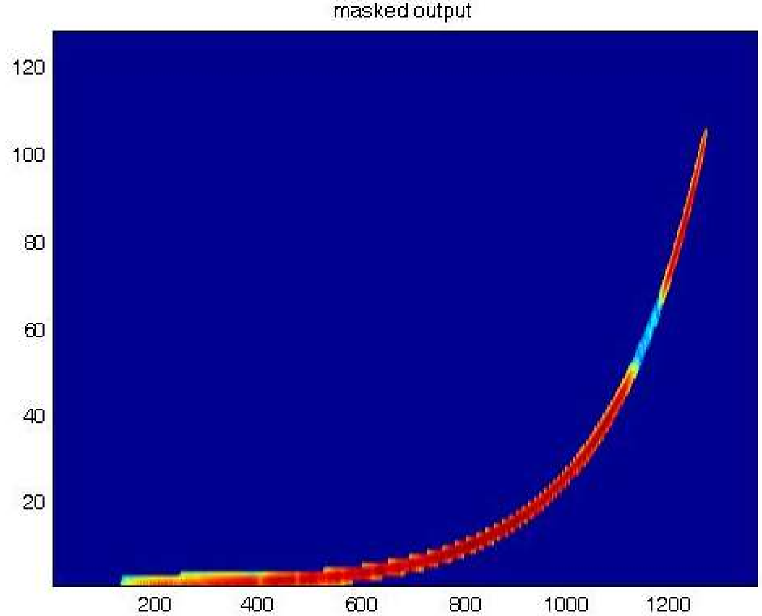


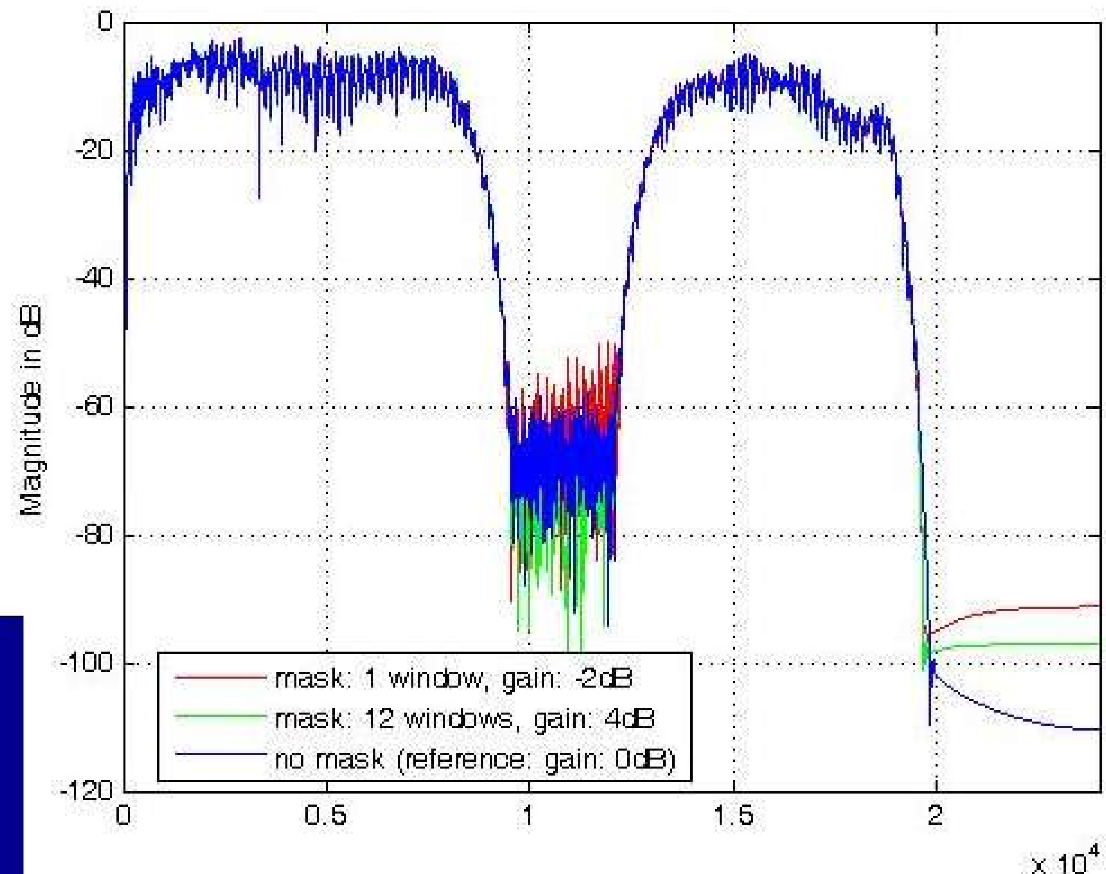
1155 systems with overlapping of 12 systems (5 sec total)

→ ***14 Minutes*** vs. 29 Min.

Increasing SNR with Gabor Multiplier







Improvement in this example:

→ ***+4dB***

Summary - Comparison

- Direct Method, PIE
- Maximum Length Sequence:
  - highest possible SNR
  - sensitive to distortions
- Exponential Sweeps:
  - separation of nonlinear distortions
  - simultaneous measurement of THD
  - measurements of multiple systems
  - high SNR (-3dB compared to MLS)
  - sensitive to transients