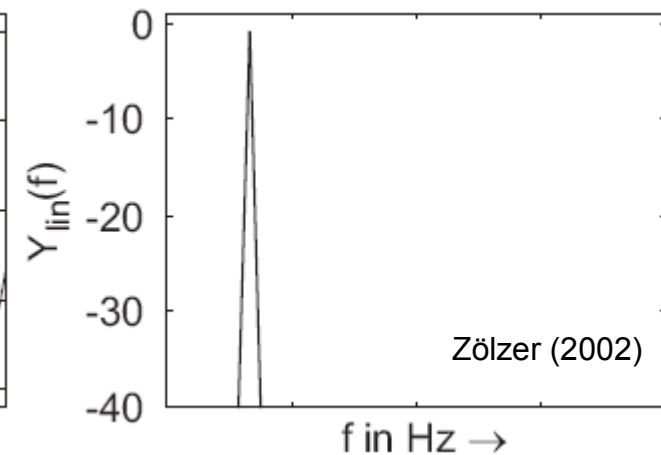
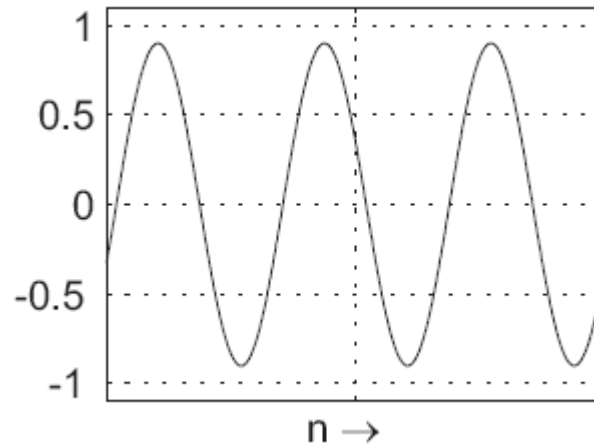
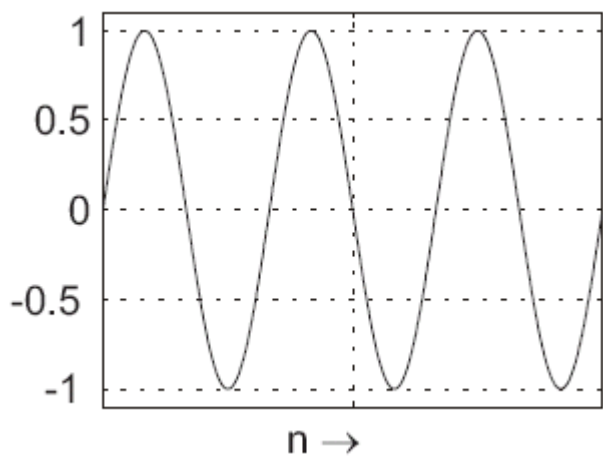


# Inhalt

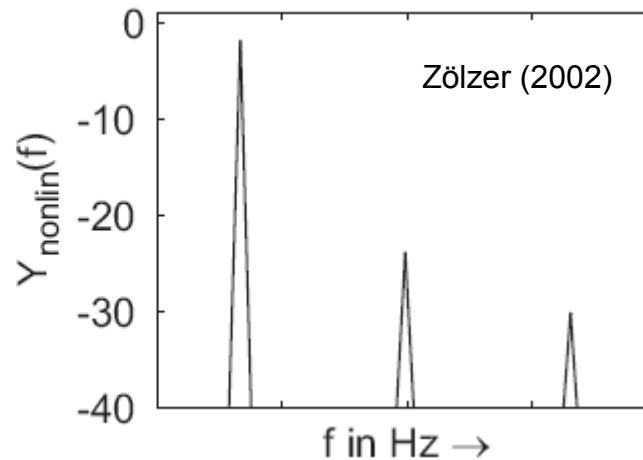
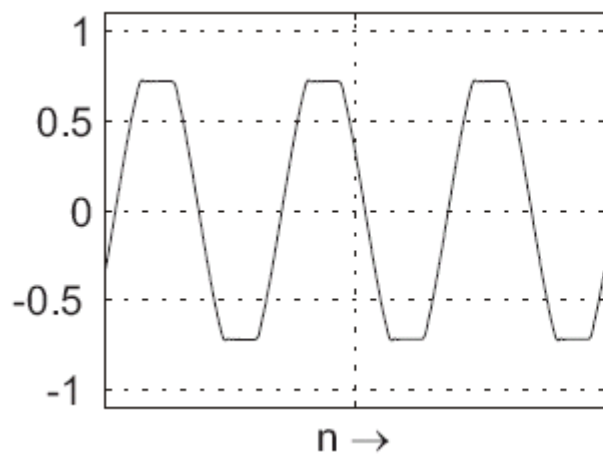
- Nichtlineare Systeme
- Voltera Reihenexpansion
- Statische quasi-nichtlineare Systeme
- Dynamikprozessoren
  - Kontrollparameter
  - Anwendungen
- Frequenzabhängige nichtlineare Systeme
- Signalverfremdung
- Messparameter

# Allgemein

- **LTI:**  $A \sin(\omega T) \longrightarrow B \sin(\omega T + \varphi) \longrightarrow H(z)$



- **Nichtlinear TI:**  $A \sin(\omega T) \longrightarrow B \sin(\omega T + \varphi)$



$$+ \sum_n B_n \sin(n \cdot \omega T + \varphi_n)$$

$n \in \mathbb{N}$

# Definition

- Lineare Systeme:

$$T \{a \cdot x(t) + b \cdot y(t)\} = a \cdot T \{x(t)\} + b \cdot T \{y(t)\}$$

- Nichtlineare Systeme:

- Systeme, die die obere Gleichung NICHT erfüllen
- Keine Einheitliche Definition für alle nichtlineare Systeme!
- Verschiedene Systemklassen: viele verschiedene Definitionen

# Polynomiale Systeme

- Klasse der polynomialen Systeme:

$$y[n] = \sum_{i=0}^p f_i \{ x[n], x[n-1], \dots, x[n-N], \\ y[n-1], y[n-2], \dots, y[n-M] \}$$

- $f_i\{\}$  ist ein Polynom der i-ten Ordnung
- Superposition verschiedener Polynome
- Polynomiales System des Grades  $p$

# Bilinearer Filter

- Idee:  $x, y, x \cdot y \rightarrow y[n]$

$$y[n] = \sum_{i=0}^{N_1} a_i x[n-i] + \sum_{j=1}^{N_2} b_j y[n-j] + \sum_{i=0}^{N_3} \sum_{j=0}^{N_4} c_{ij} x[n-i] y[n-j]$$

- Rekursiver Anteil
- Erweiterung eines linearen IIR-Filters

# Quadratischer Filter

- Idee:  $x, x^2 \rightarrow y[n]$

$$y[n] = \sum_{i=0}^{N_1} h_1 x[n-i] + \sum_{i=0}^{N_2} \sum_{j=0}^{N_2} h_2[i, j] \cdot x[n-i] \cdot x[n-j]$$

- System 2-ter Ordnung
- Keine Rückkopplung
- Erweiterung eines linearen FIR-Filters
  - $h_1$  ... Impulsantwort
  - $h_2$  ... Kernfunktion

# Homogene Gleichung

- Polynomiales System:

$$y[n] = \sum_{i=0}^p f_i \{ x[n], x[n-1], \dots, x[n-N], \\ y[n-1], y[n-2], \dots, y[n-M] \}$$

- Übergang:  $y = f \{ x, y \} \rightarrow y = f \{ h, x \}$

- Homogene Gleichung:

$$f_i \{ \} = \sum_{v_1} \sum_{v_2} \dots \sum_{v_N} h[v_1, v_2, \dots, v_N] x[n-v_1] x[n-v_2] \dots x[n-v_N]$$

Kernfunktion

# Volterra Series Expansion

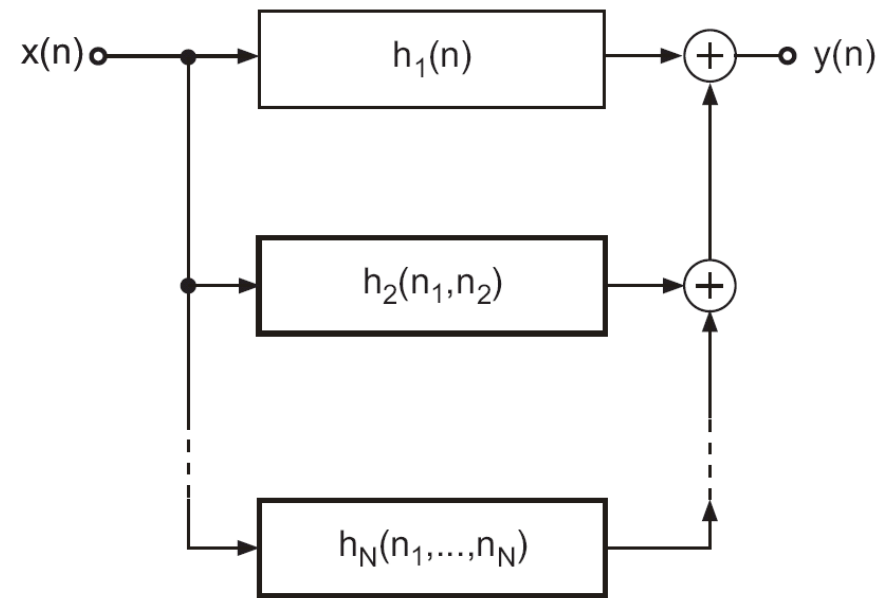
- $$y[n] = \sum_{i=1}^p f_i[n]$$

$$= \sum_{v_1=0} h_1[v_1]x[n-v_1] +$$

$$\sum_{v_1=0} \sum_{v_2=0} h_2[v_1, v_2]x[n-v_1]x[n-v_2]$$

$$+ \dots + \sum_{v_1=0} \sum_{v_2=0} \dots \sum_{v_p} h_p[v_1, v_2, \dots, v_p]$$

$$x[n-v_1]x[n-v_2] \dots x[n-v_p]$$



Zölzer (2002)

- Für leicht nichtlineare Systeme besonders gut geeignet (Ordnung  $p$  ist klein)



# Volterra System: Factored Kernels

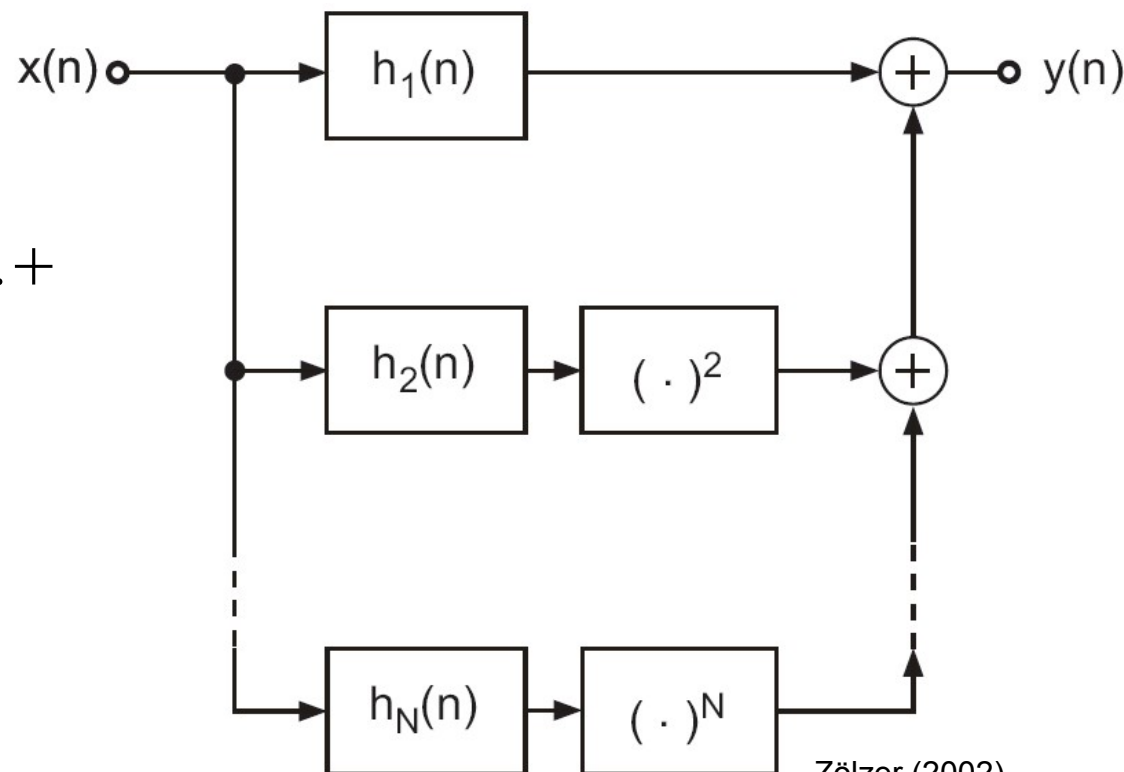
- Faktorisierung der Kernfunktionen:

$$h_i[v_1, v_2, \dots, v_i] = h_i^f[v_1] \cdot h_i^f[v_2] \cdot \dots \cdot h_i^f[v_i]$$

$$y[n] = \sum_{v_1=0} h_1^f[v_1] x[n-v_1] +$$

$$\left( \sum_{v_2=0} h_2^f[v_2] x[n-v_2] \right)^2 + \dots +$$

$$\left( \sum_{v_N=0} h_N^f[v_N] x[n-v_N] \right)^N$$



# Statische nichtlineare Systeme

- Für Systeme ohne Speicher:
  - Kern  $h_i^f[v_i]$  wird zu einem Skalar:

$$h_i[n_1, n_2, \dots, n_i] = A_i \cdot \delta[n]$$

- Der Ausgang kann durch ein Polynom beschrieben werden:

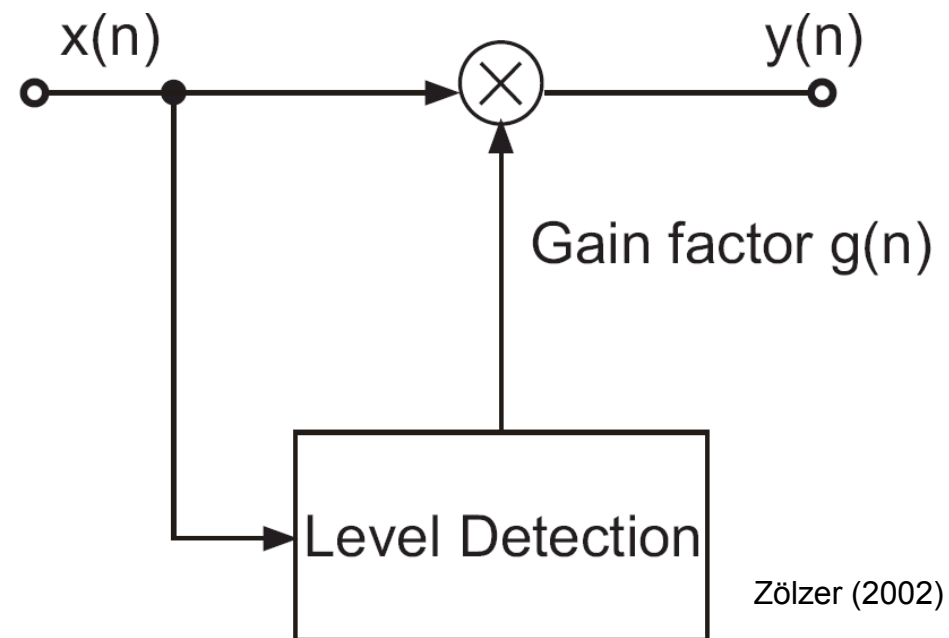
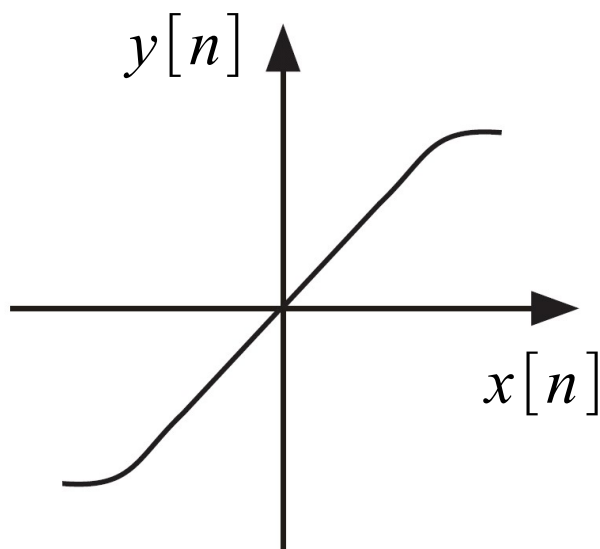
$$y[n] = \sum_{i=0} A_i x^i[n]$$

- Allgemein:  $y[n] = f(x[n])$

# Verstärkungskennlinie

- Für statische nichtlineare Systeme genügt eine Kennlinie:

$$y[n] = f(x[n])$$



# Dynamikprozessoren

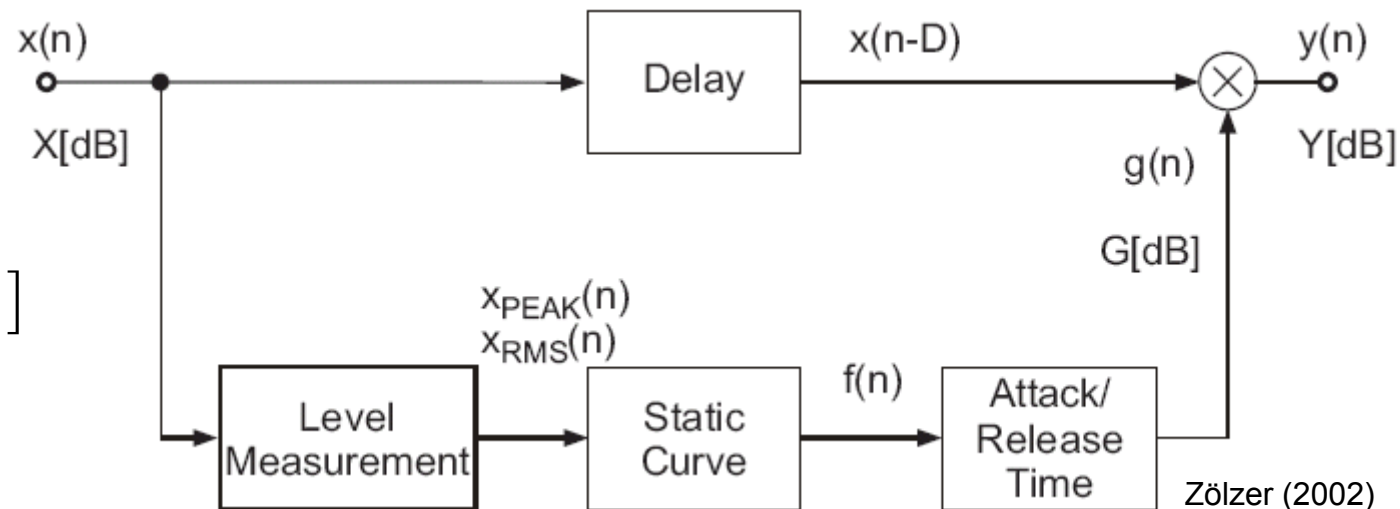
- Veränderung der Einhüllenden laut Parametrierung
- Harmonische Verzerrung so gering wie nur möglich

$$y[n] = f(x[n])$$

$$y[n] = x[n] \cdot g[n]$$

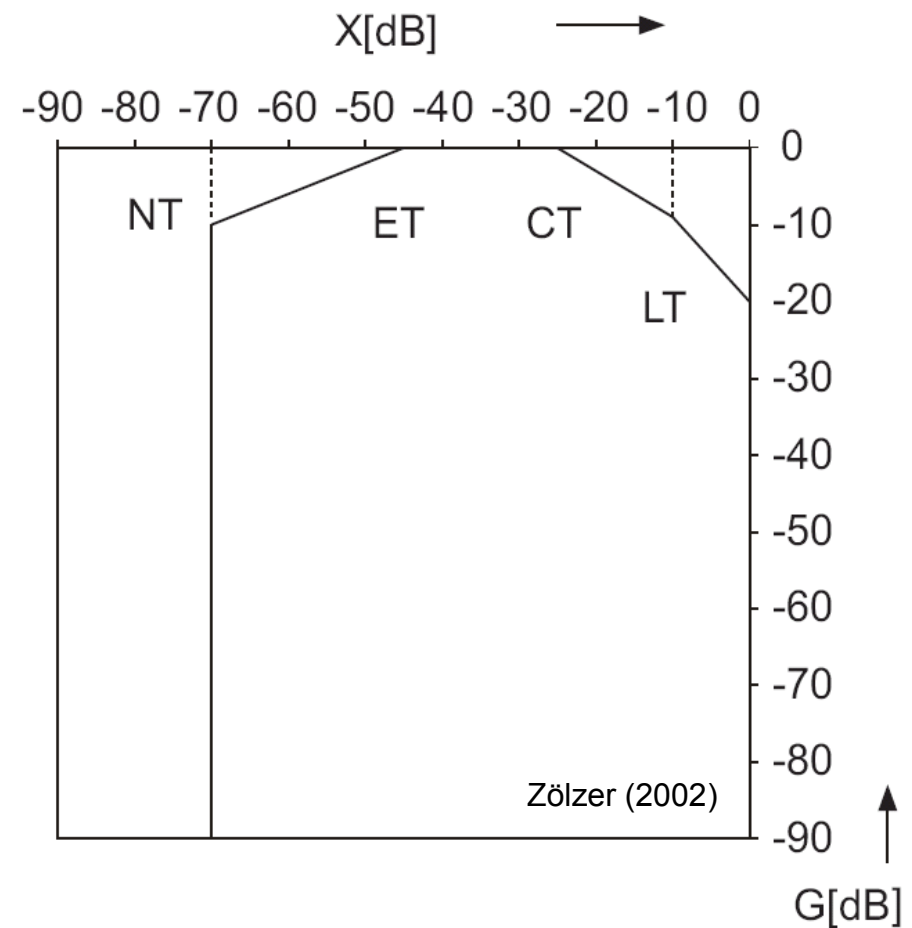
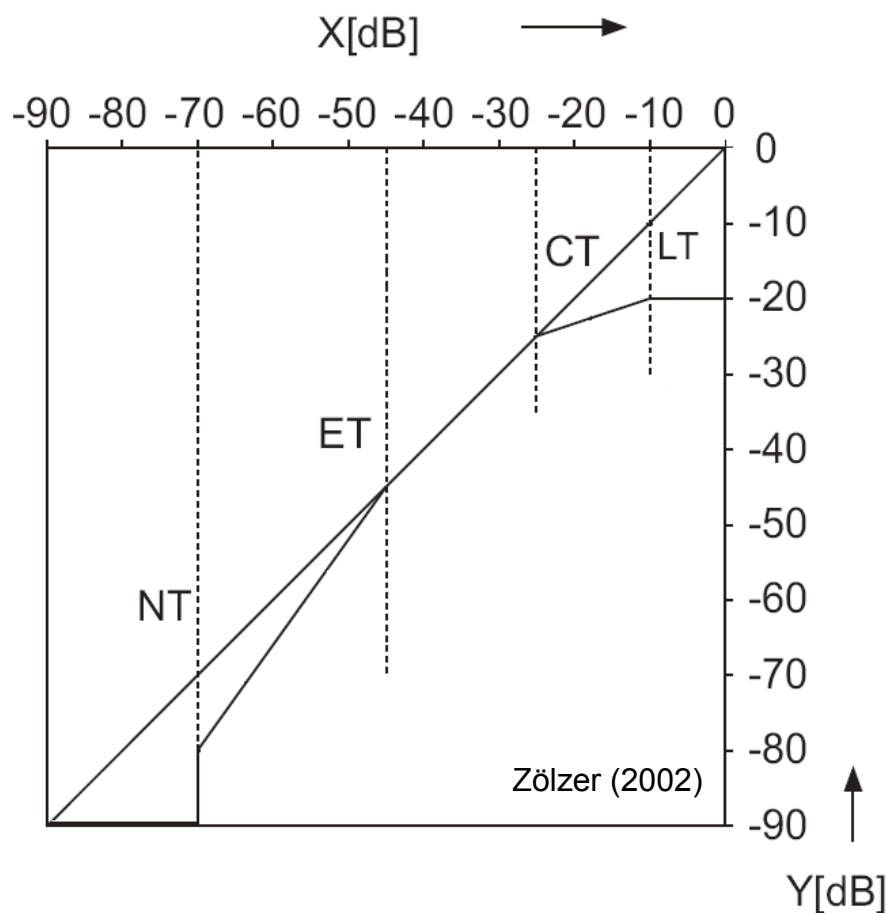
- in dB:

$$Y[N] = X[N] + G[N]$$



# Verstärkungskennlinie

- Zusammenhang:  $Y[N]$  und  $X[N]$



# Verstärkungskennlinie - Parameter

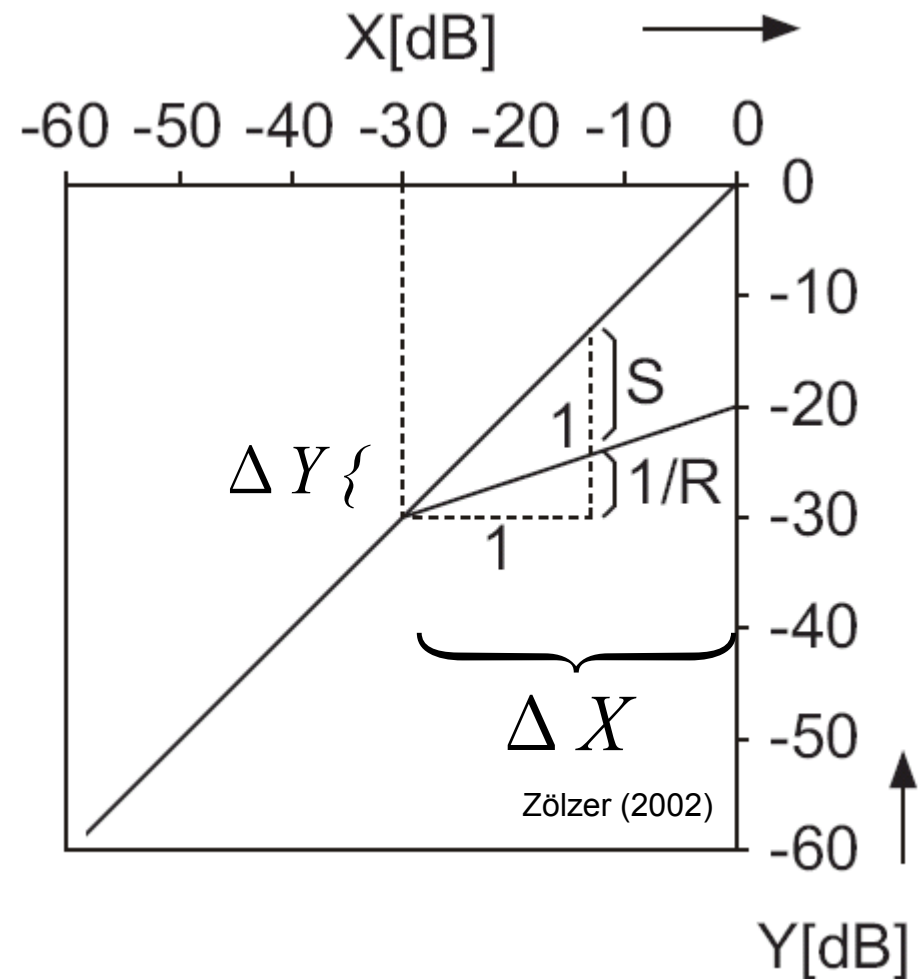
- Kompressionsfaktor:

$$R = \frac{\Delta X}{\Delta Y}$$

- Steigung (Slope Factor):

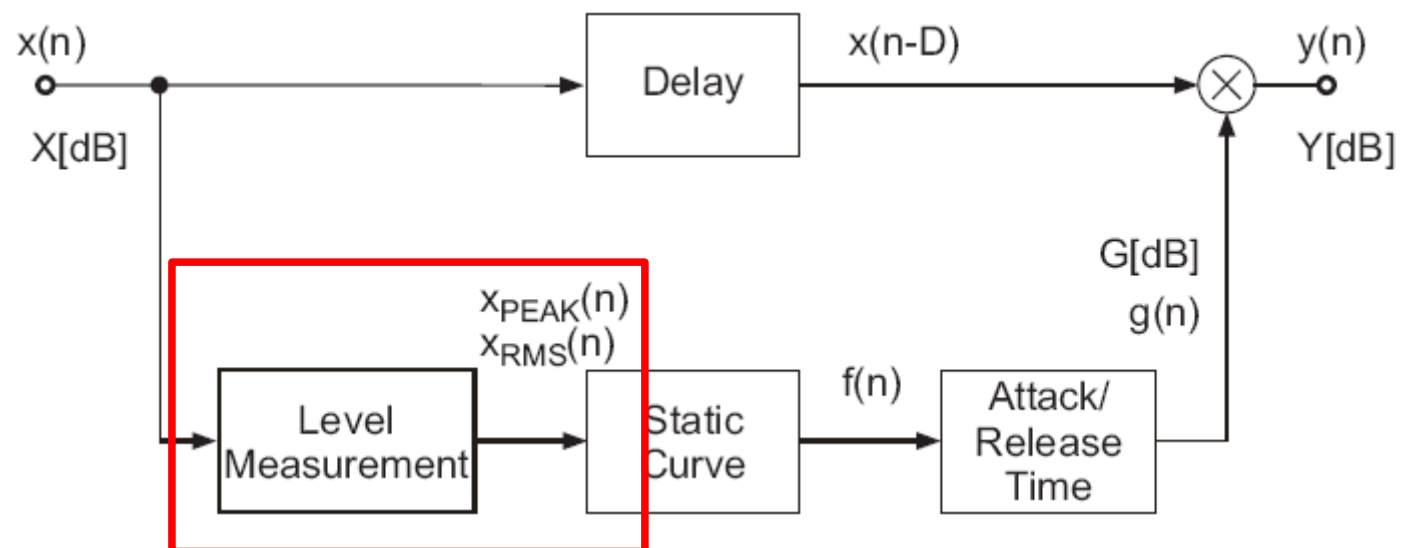
$$S = 1 - \frac{\Delta Y}{\Delta X}$$

$$S = 1 - \frac{1}{R} \quad R = \frac{1}{1 - S}$$



# Control Parameter

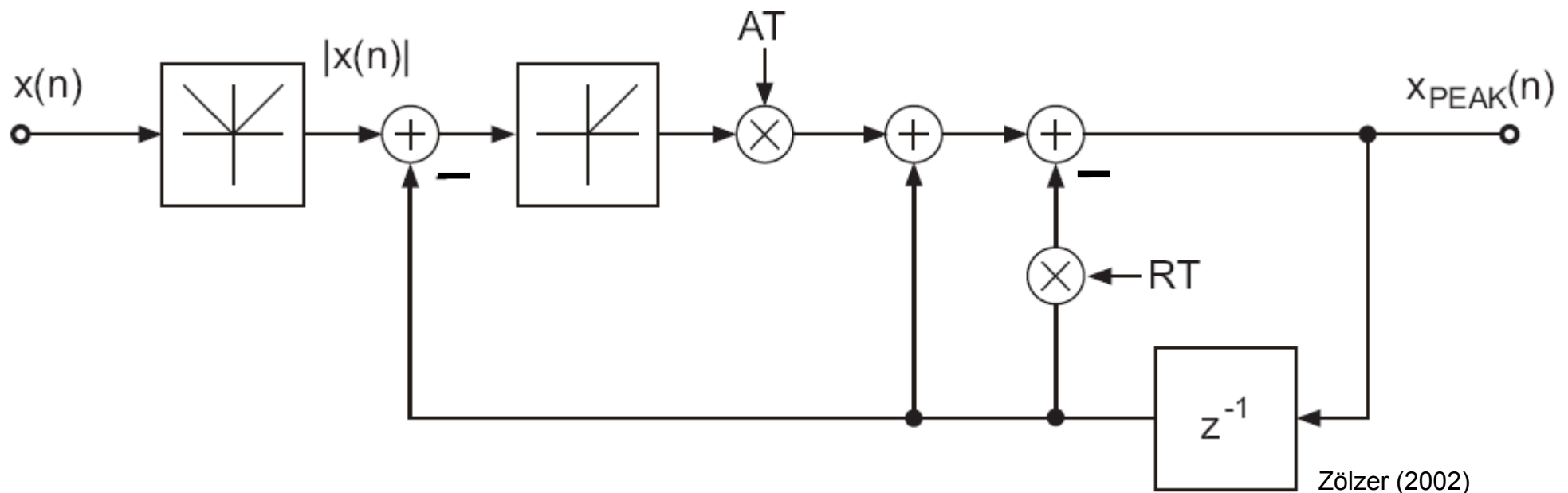
- Einhüllendenverfolgung (Envelope Follower):
  - Spitzenwert
  - Effektivwert (RMS)



# Spitzenwert

- Attack time  $t_{AT}$ :  $AT = 1 - e^{-2.2T_s/t_{AT}}$
- Release time  $t_{RT}$ :  $RT = 1 - e^{-2.2T_s/t_{RT}}$

$T_s \dots$  Abtastperiode

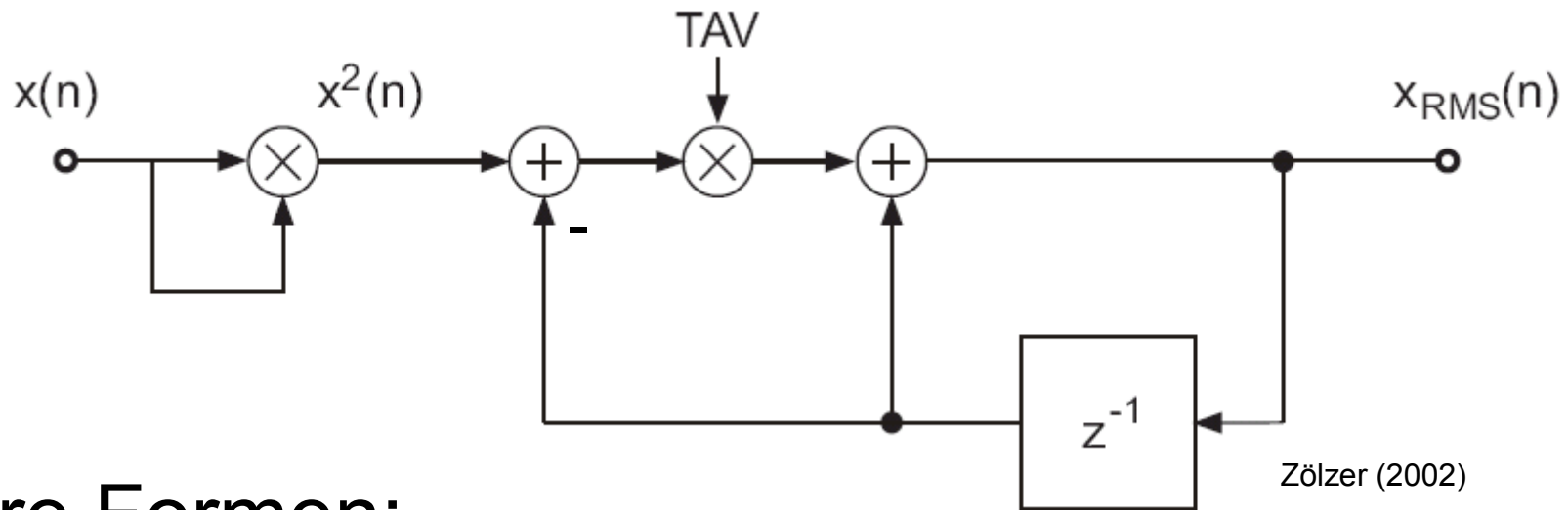




# Effektivwert (RMS)

- Average time  $t_{AVR}$ :  $TAV = 1 - e^{-2.2T_S/t_{AVR}}$

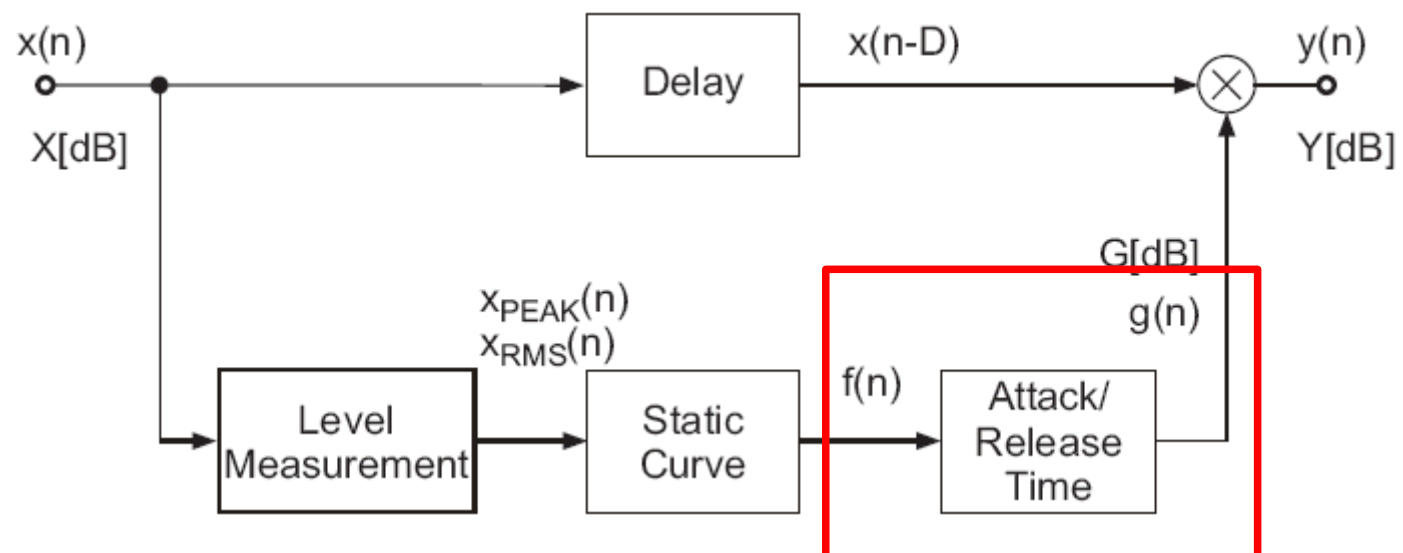
$T_S \dots$  Abtastperiode



- Andere Formen:
  - Cepstrum
  - Hilbert-Transformation

# Kontrollparameter

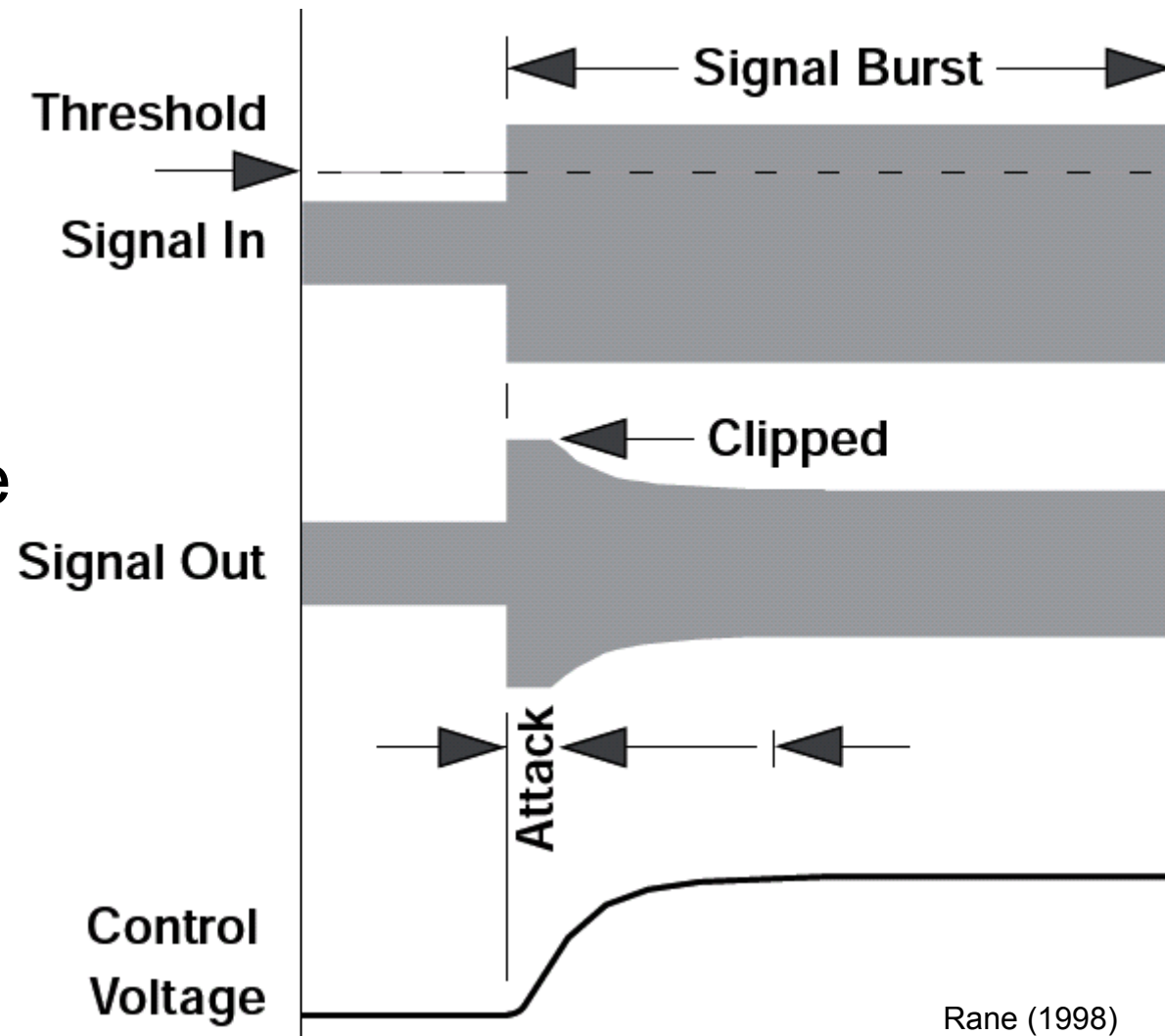
- Dynamik der Kennlinie:
  - Attack Time
  - Release Time



Zölzer (2002)

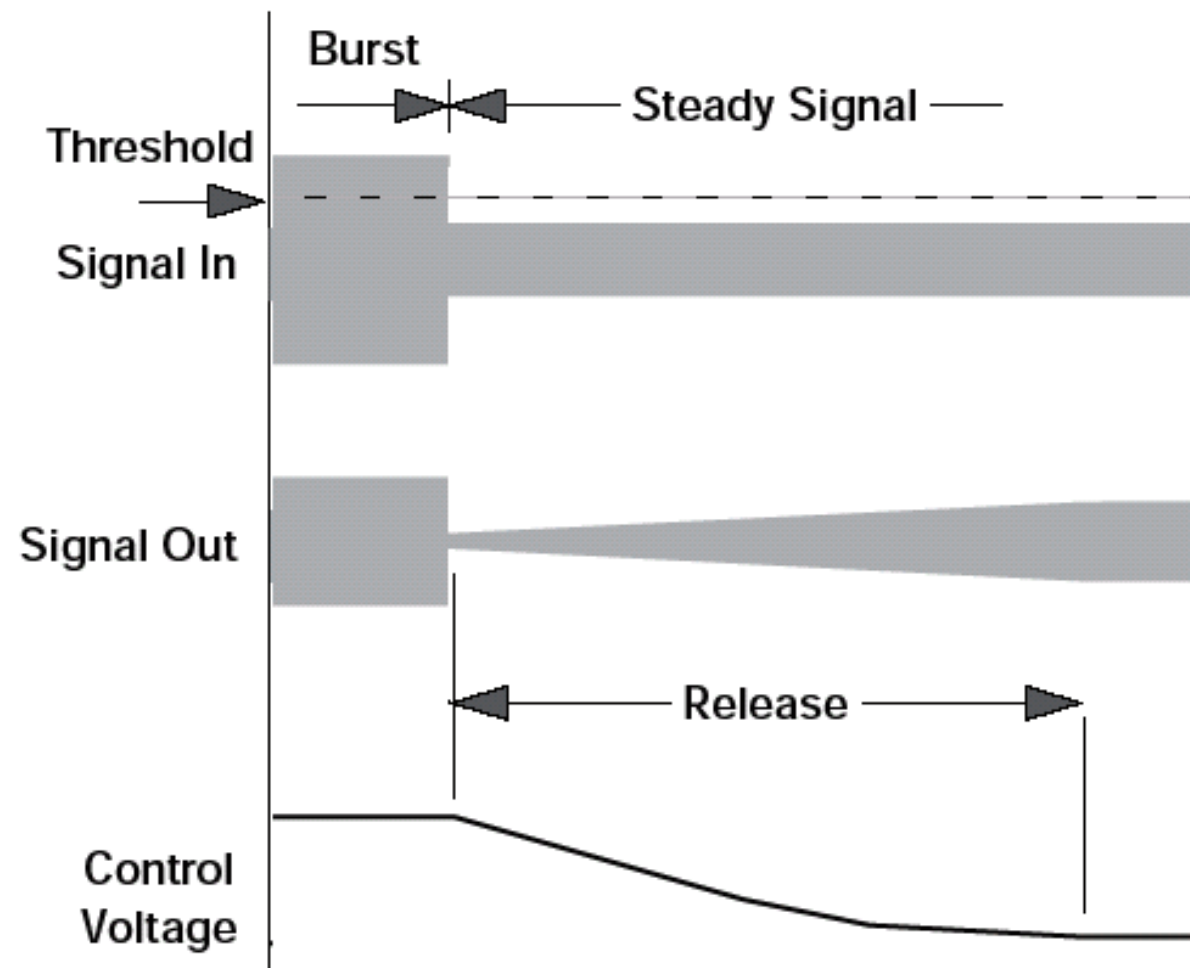
# Attack Time

- Attack Time  $t_{AT}$
- Zu kurz:
  - Dynamikfluktuation
  - Verzerrungen tieffrequenter Signale
- Zu lang:
  - Clipping-Gefahr
- Bereich: 0 - 100ms



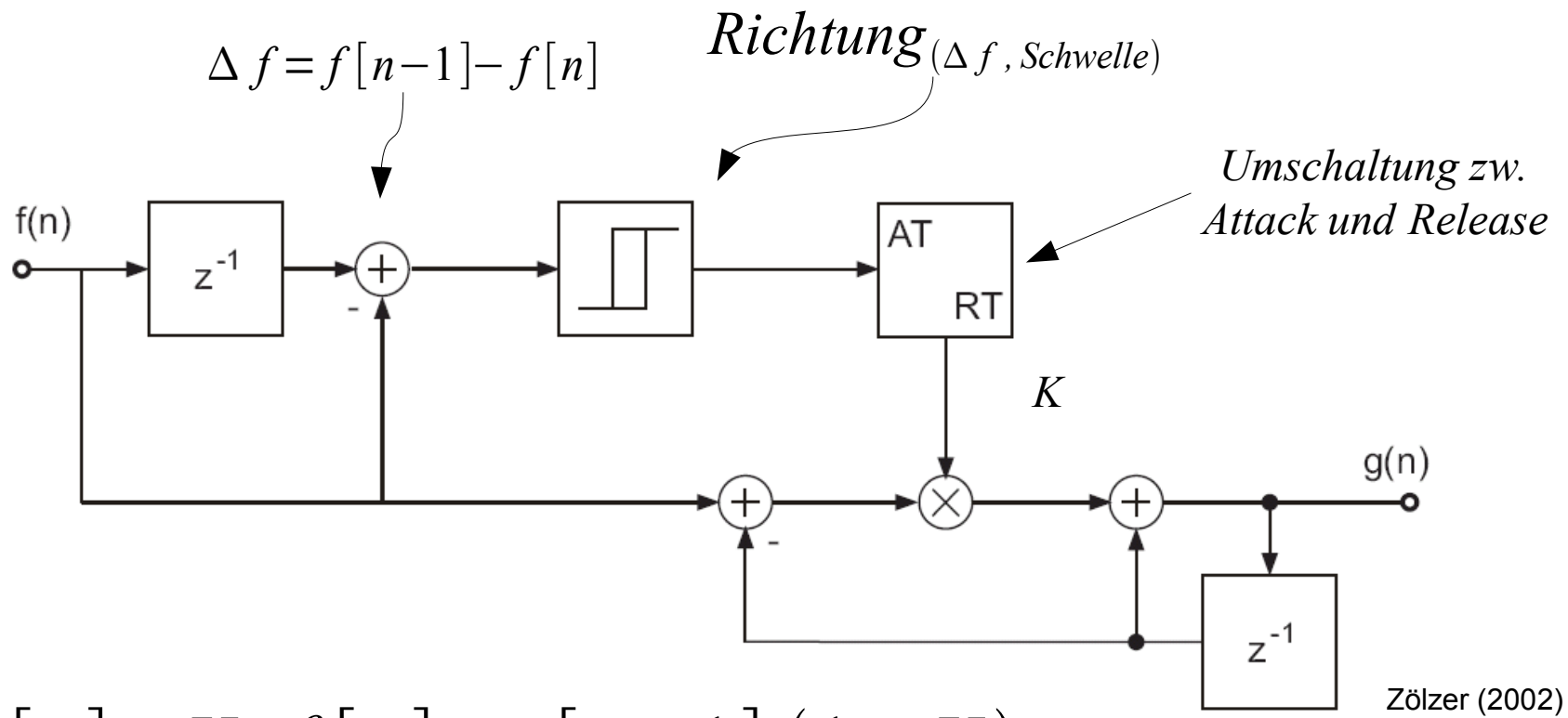
# Release Time

- Release Time  $t_{RT}$
- Zu schnell:
  - Chopping
- Zu langsam:
  - Atmen/Pumpen
- Bereich: 100ms - 3s



# Attack / Release Time Control

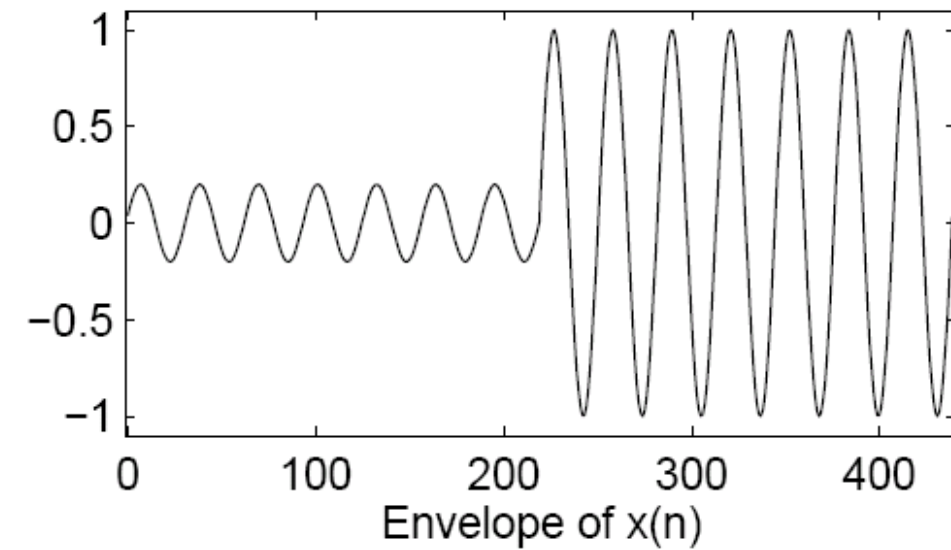
- Struktur:



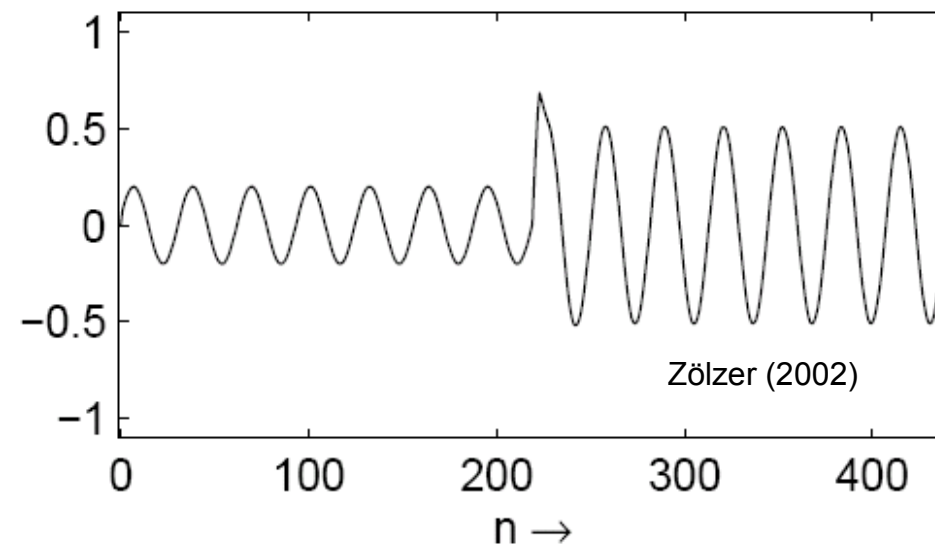
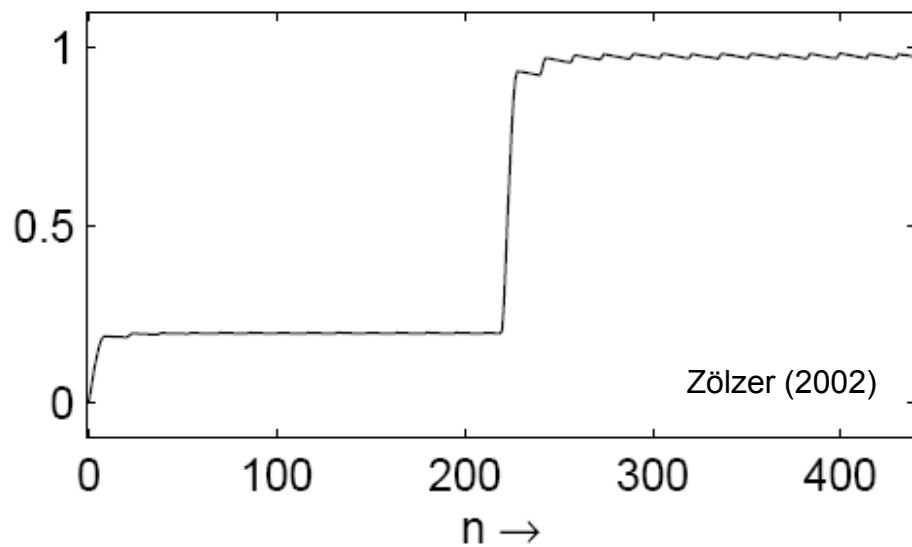
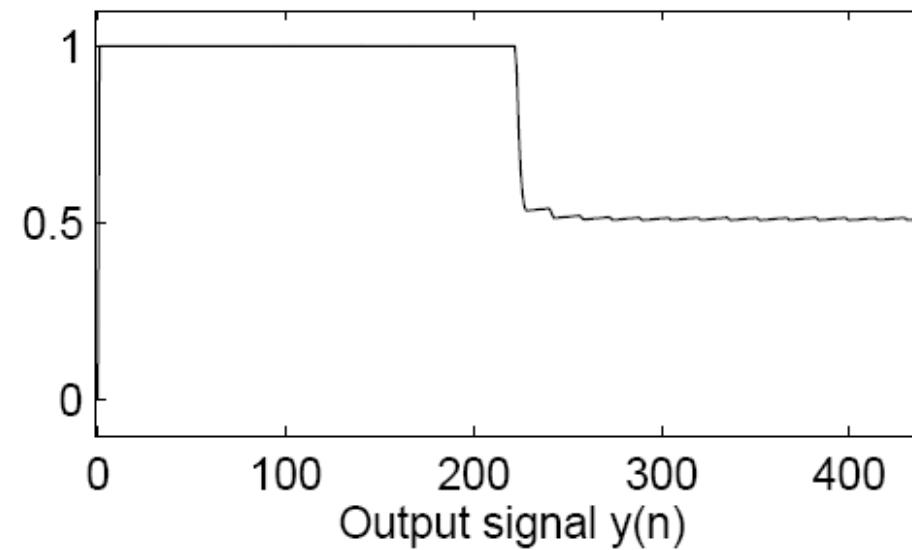
$$g[n] = K \cdot f[n] + g[n-1] \cdot (1 - K)$$

# Dynamikprozessoren

Input signal  $x(n)$



Gain factor signal  $g(n)$



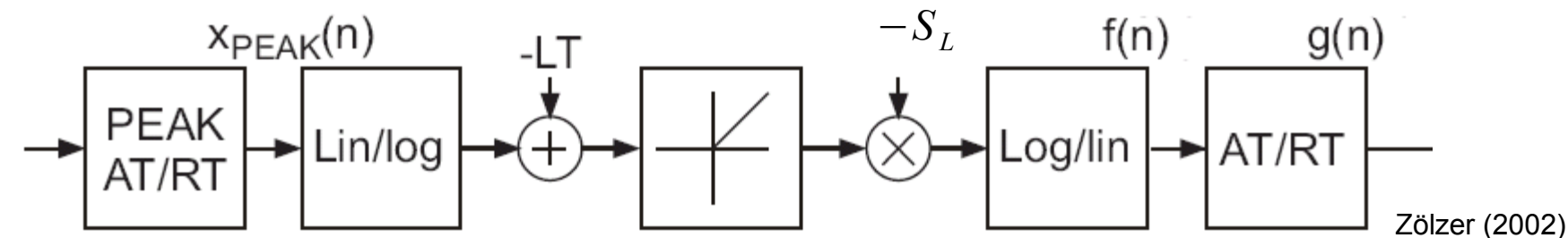
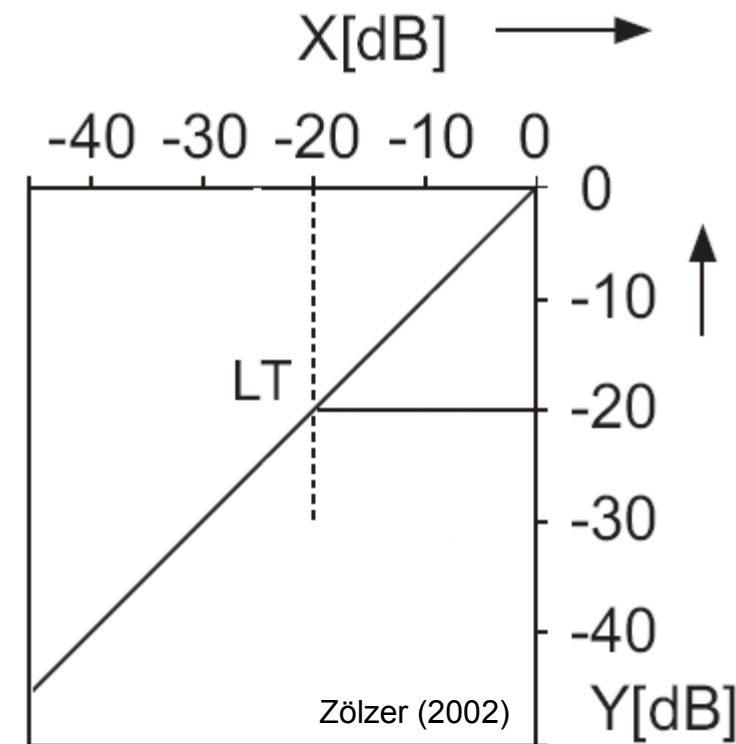
# Limiter

- Möglichst keine Dynamikveränderung
- Kontrolle der Spitzenwerte
- Parameter:

$$S_L = 1 \quad (R_L = \infty)$$

$$F = -S_L \cdot (X - LT)$$

- Side chain:



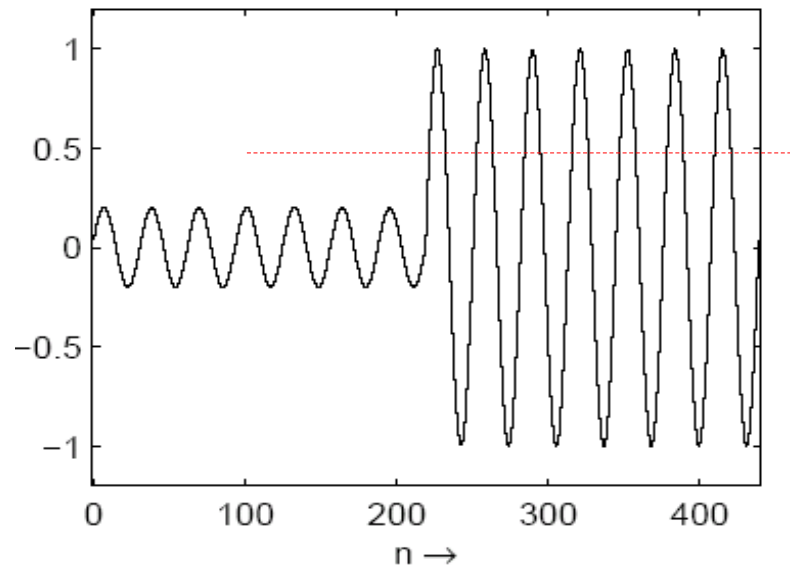
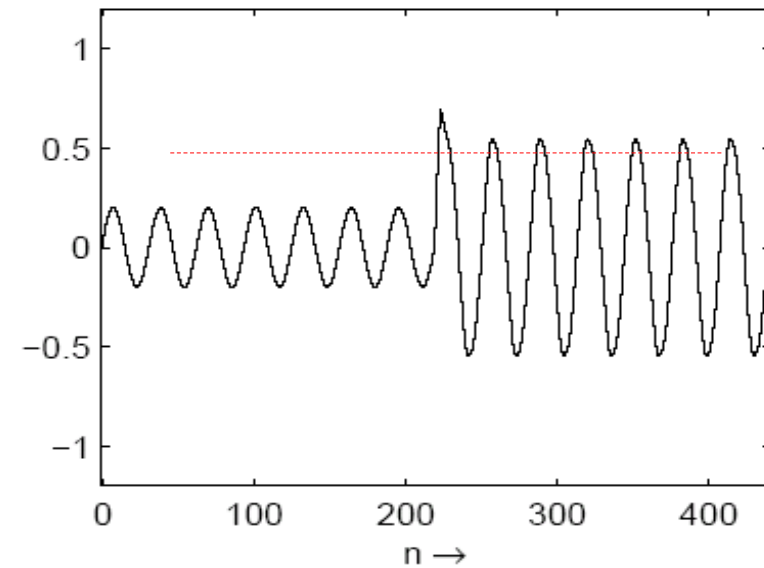
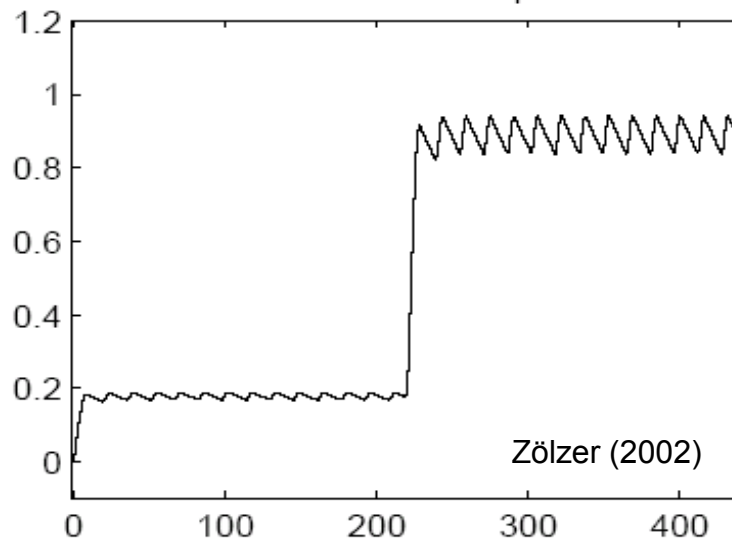
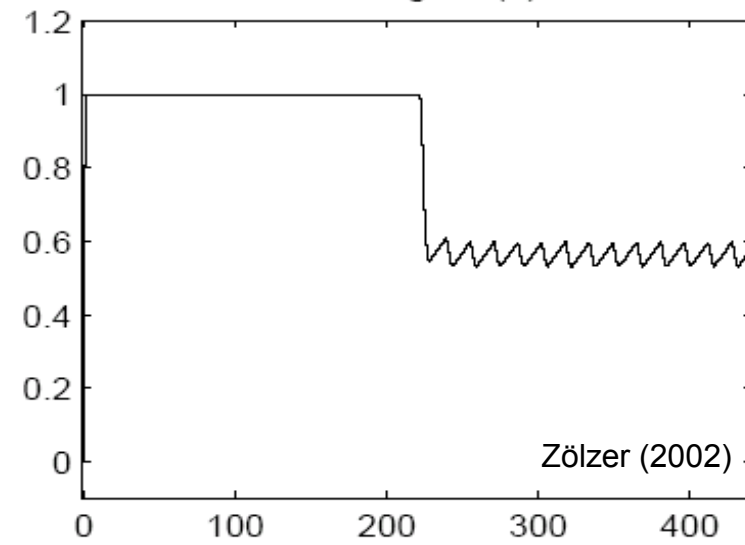
# Limiter

$$Thr = 0.5$$

$$t_{RT} = 5\text{ms}$$

$$t_{AT} = 100\ \mu\text{s}$$

$$f_s = 44.1\text{kHz}$$

Input signal  $x(n)$ Output signal  $y(n)$ Filter output signal  $x_{\text{peak}}(n)$ Gain signal  $f(n)$ 



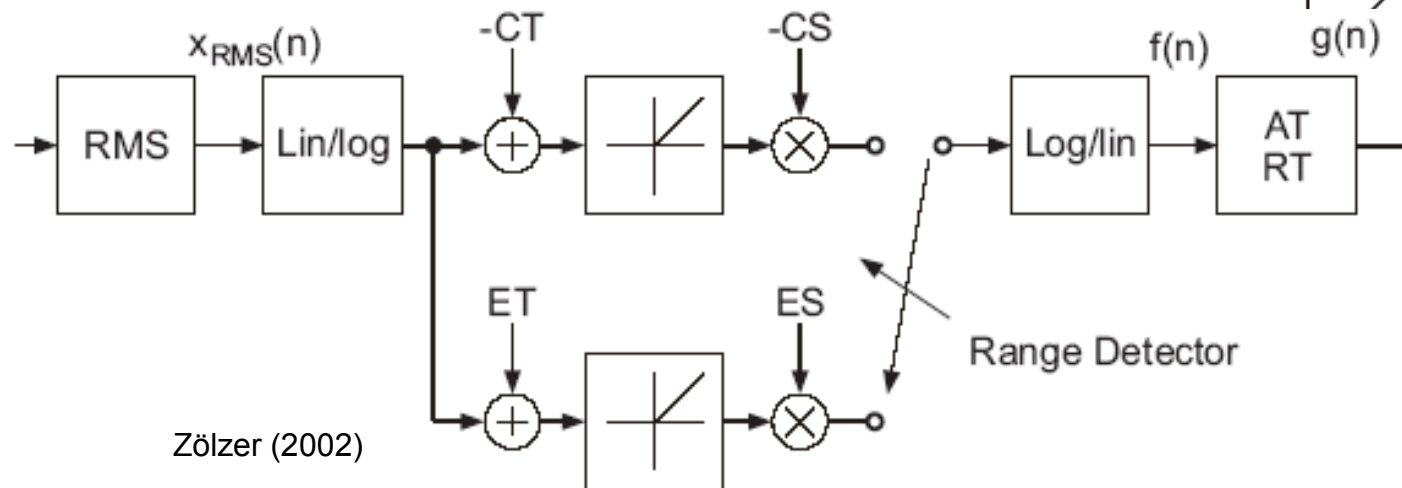
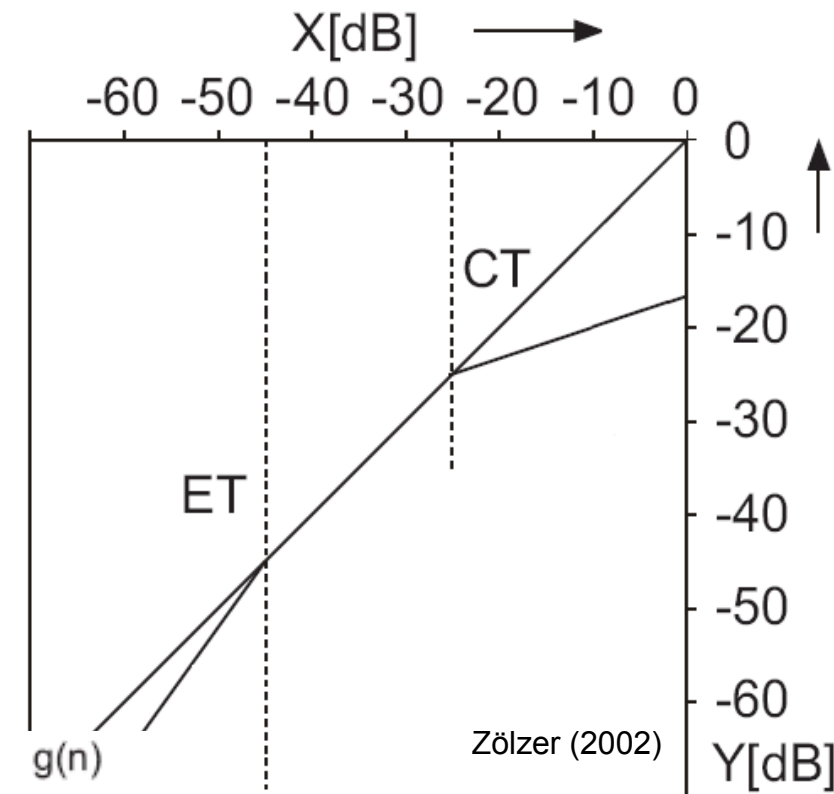
# Kompressor / Expander

- Parameter:

$$0 < S_C < 1 \quad -\infty < S_E < 0$$

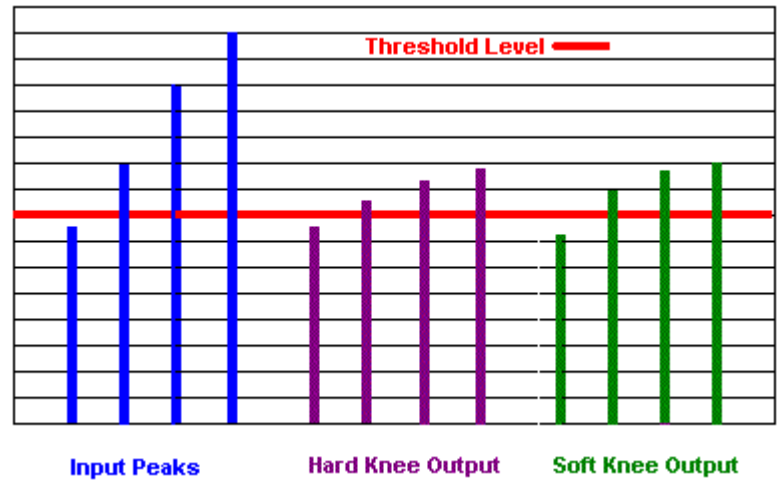
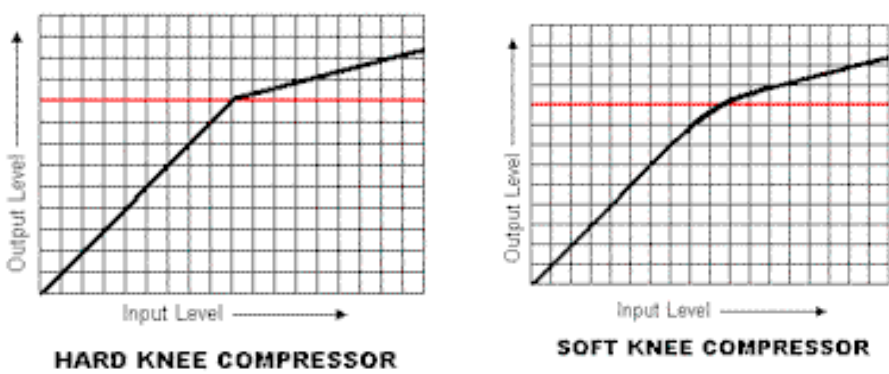
$$F_C = -S_C \cdot (X - CT)$$

$$F_E = -S_E \cdot (X - ET)$$

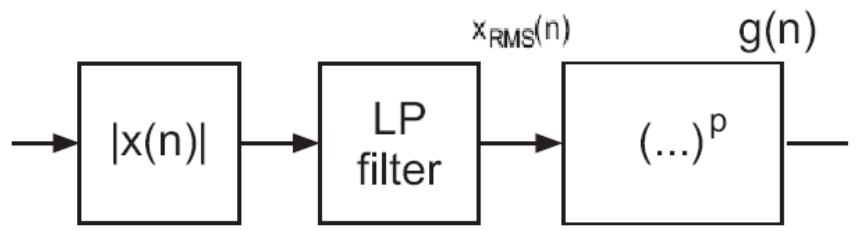
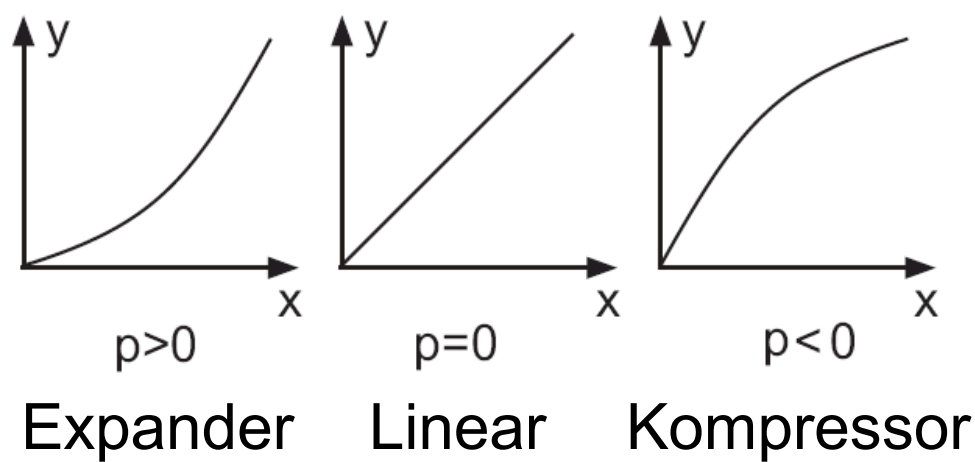


# Kompressor / Expander

- Hard vs. Soft Knee:



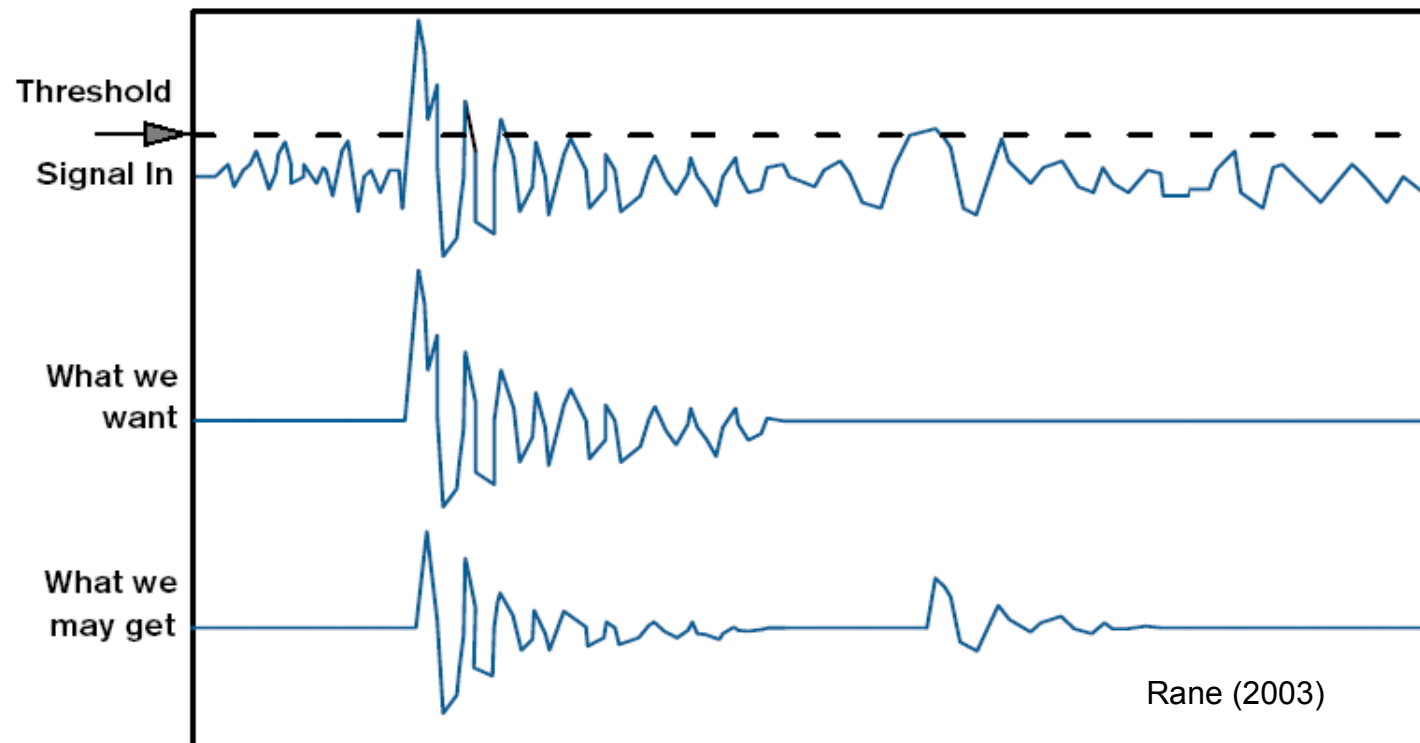
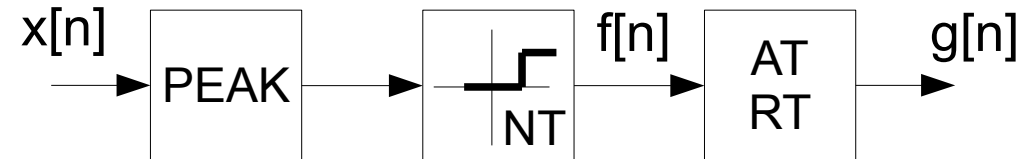
- Alternative Struktur:



# Noise Gate

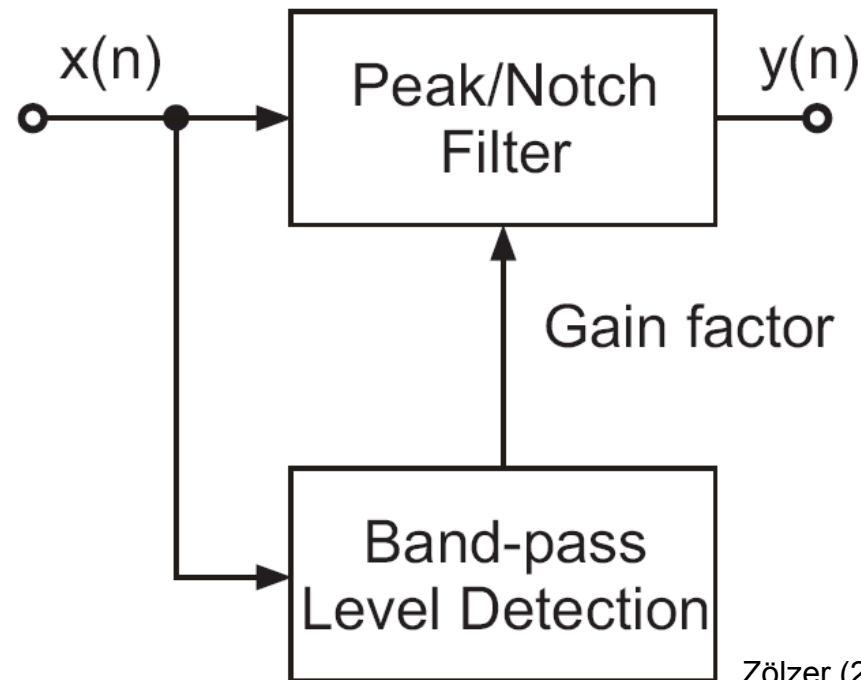
- Parameter:

- Hold time
- Depth of Cut, Floor or Ratio Control



# De-esser

- Reduktion der Zischgeräusche
- Unterschied zum Kompressor: Frequenzabhängige Verarbeitung



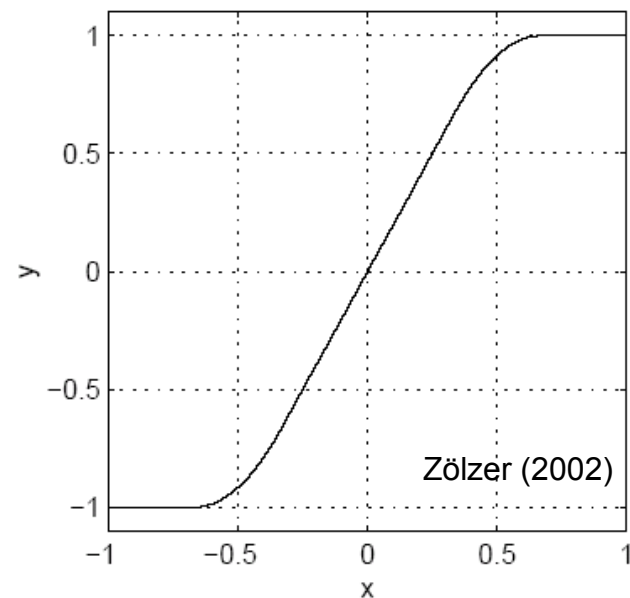
# Let's rock

- Verzerren der Signale einer elektrischen Gitarre
- Ursprünglich: Röhrenverstärker
- Jetzt: meist digital, mit simulierten Kennlinien
- Beispiele:
  - Overdrive
  - Distortion
  - Fuzz

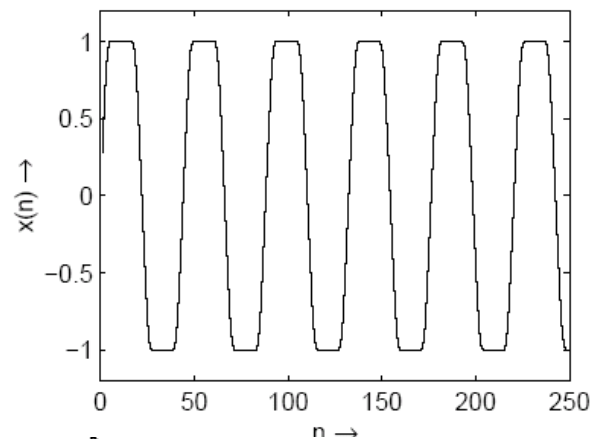
# Overdrive

- Soft Symmetrical Clipping

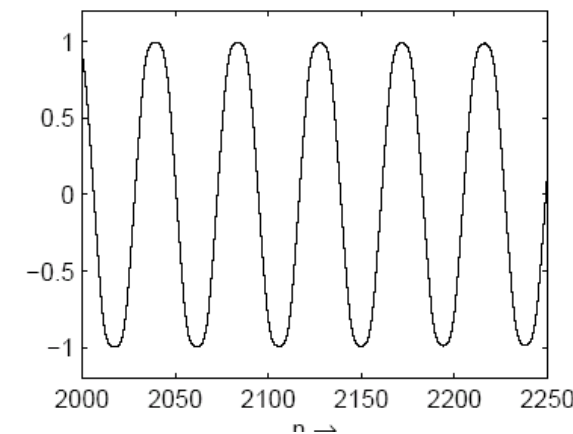
Static characteristic:  $y=f(x)$



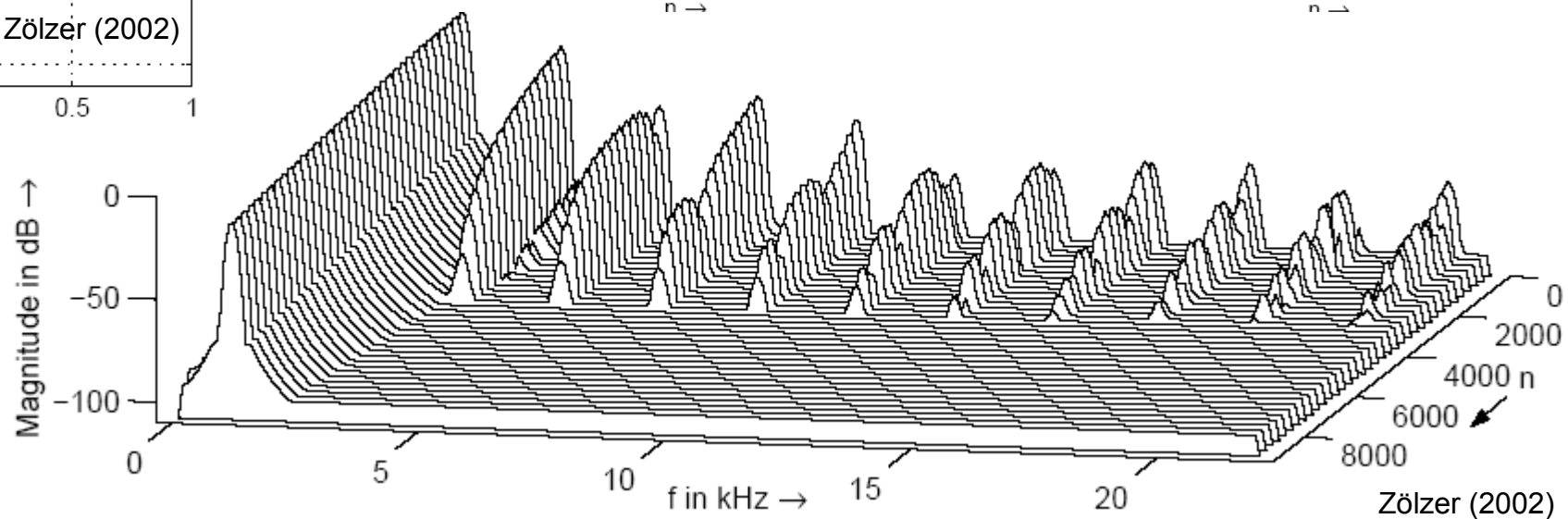
Signal  $x(n)$



Signal  $x(n)$



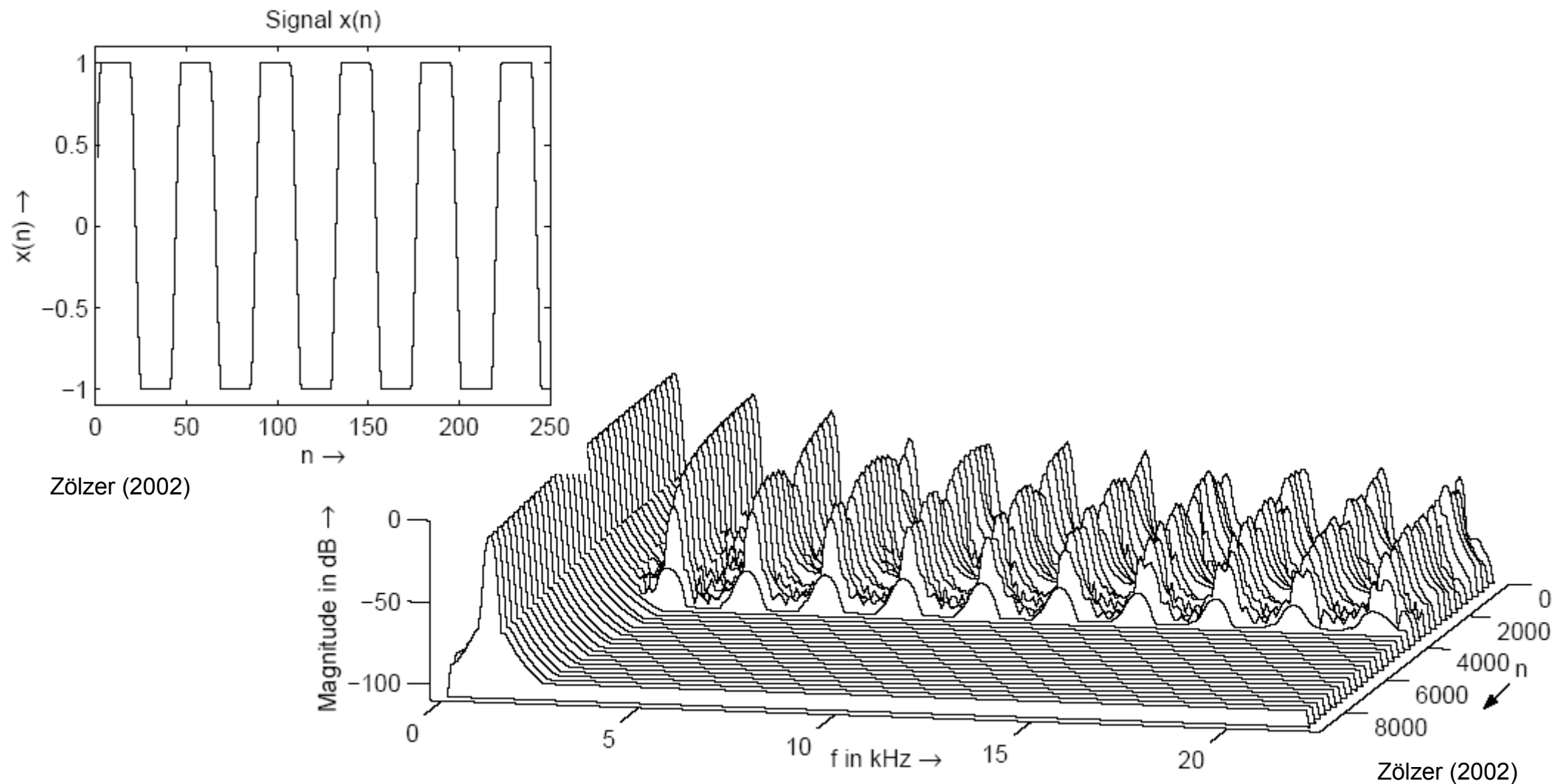
Zölzer (2002)



Zölzer (2002)

# Distortion

- Harte symmetrische Übersteuerung

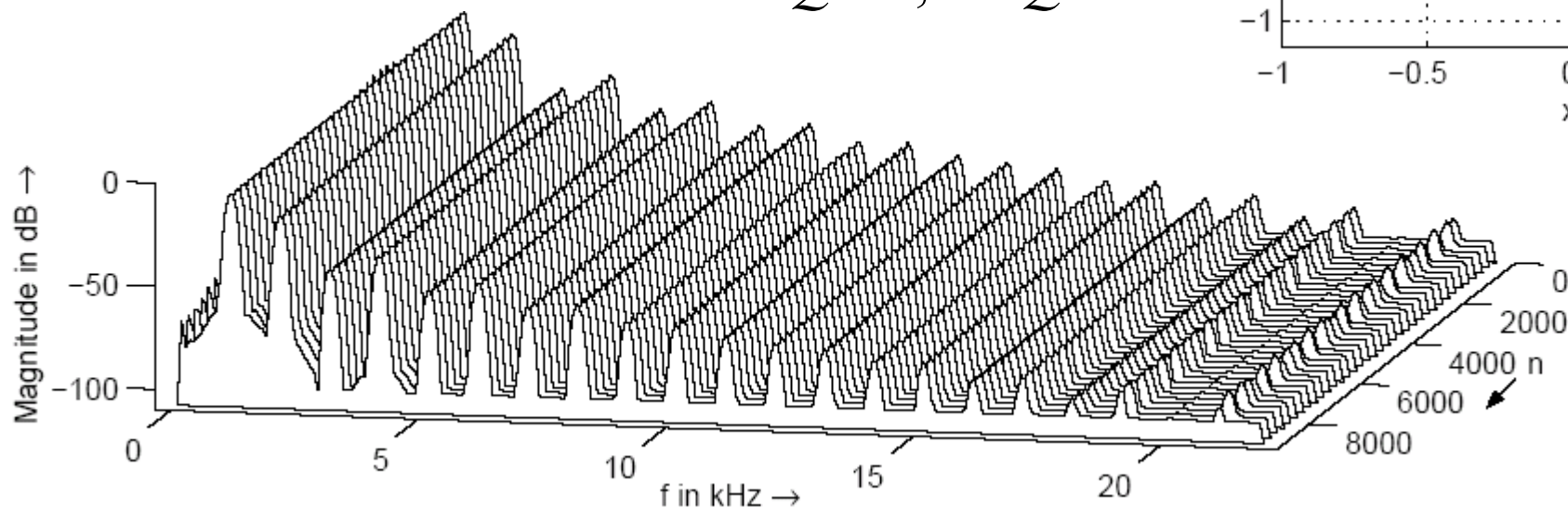
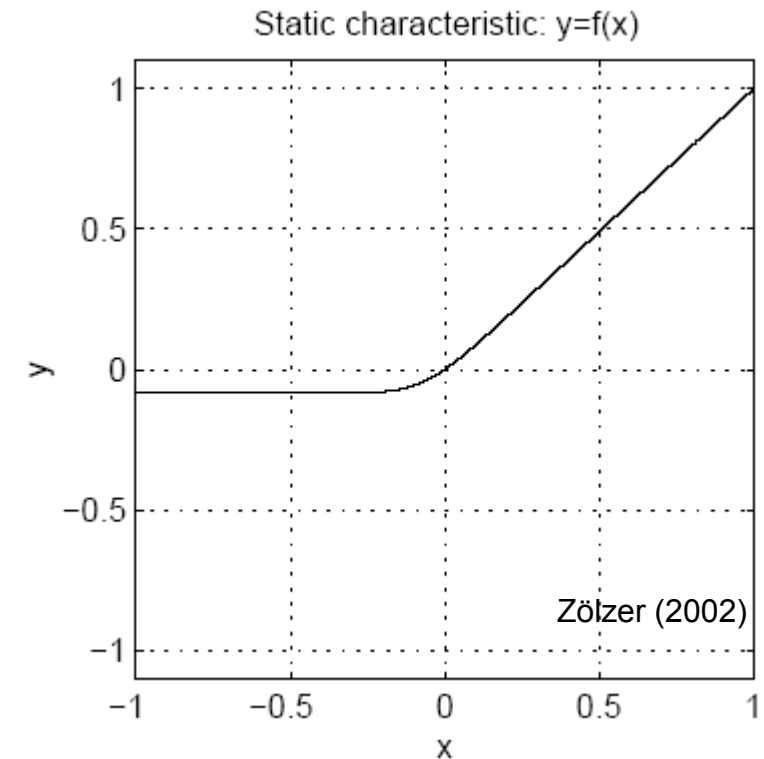


# Fuzz

- Asymmetrische Kennlinie
- Beispiel (Tube Distortion):

$$f(x) = \frac{x - Q}{1 - e^{-d(x - Q)}} + \frac{Q}{1 - e^{dQ}}$$

$$Q \neq 0, x \neq Q$$



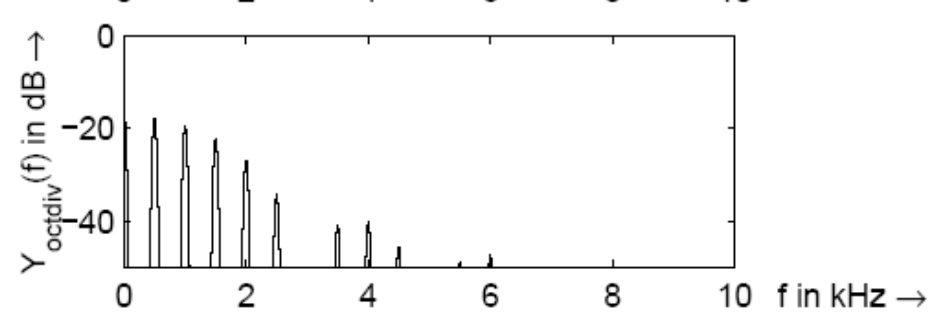
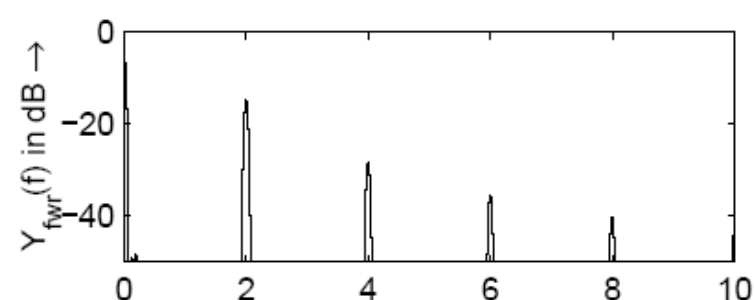
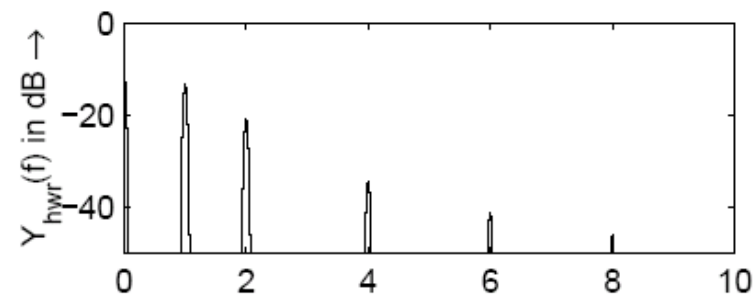
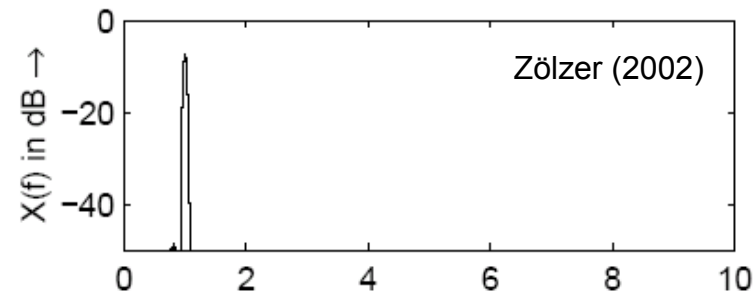
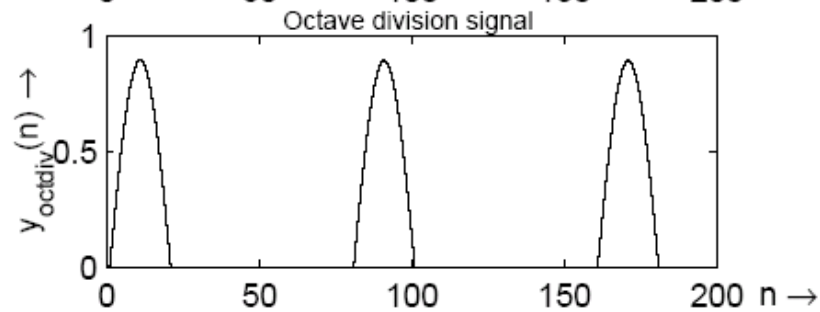
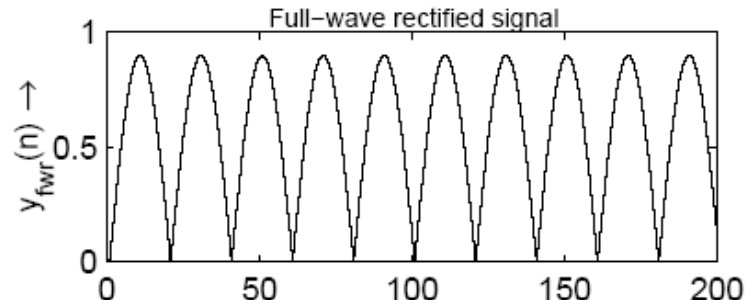
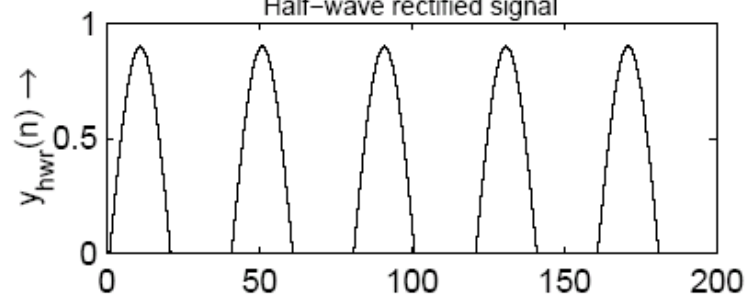
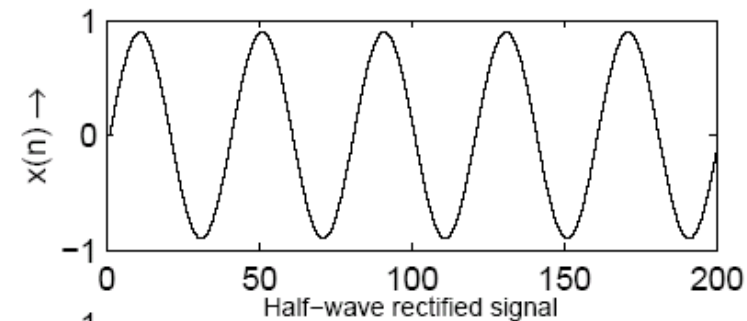
Zölzer (2002)

$$Q = -0.2$$

$$d = 8$$

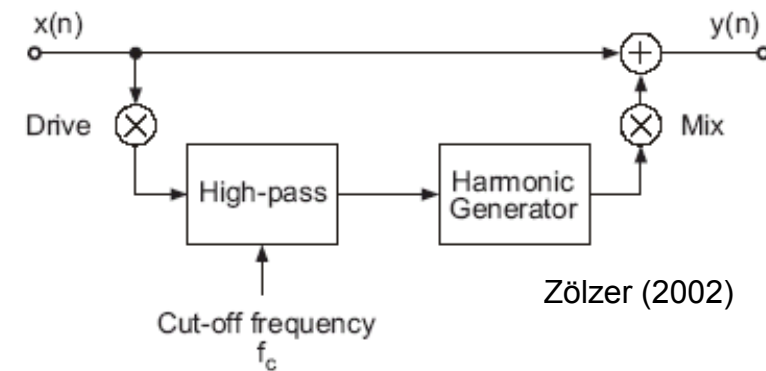
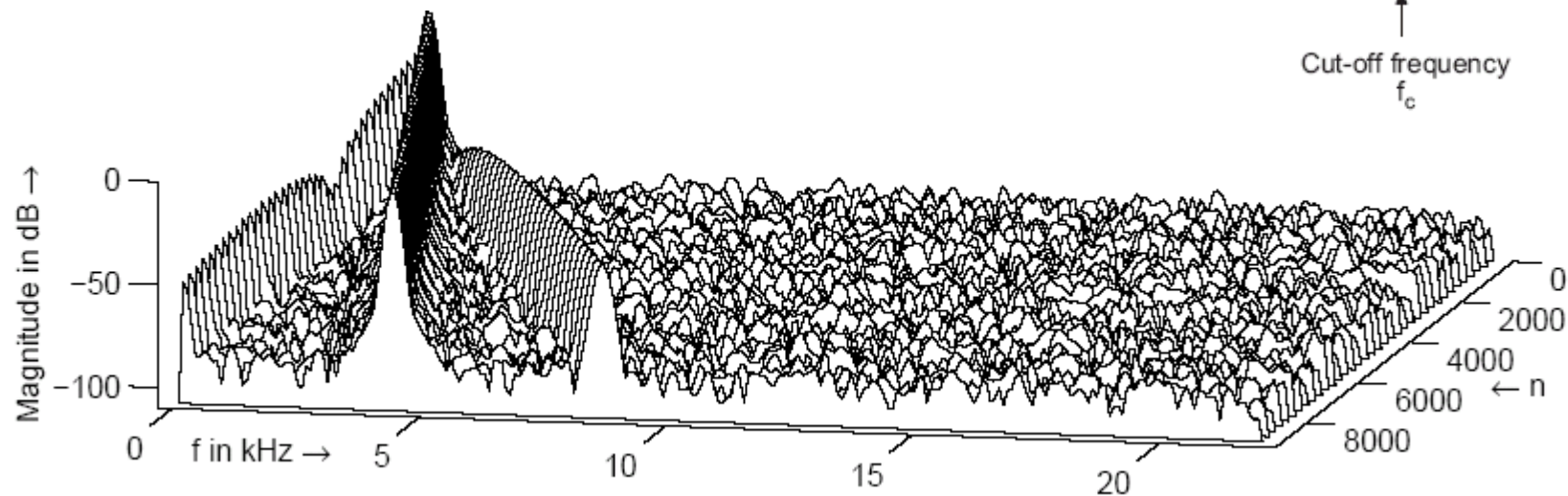


# (Sub)-Harmonische



# Exciter

- Betonung bestimmter Frequenzbereiche um Klangveränderungen hervorzurufen
- Beispiel: APHEX Aural Exciter



Zölzer (2002)

Zölzer (2002)

# Klirrfaktor k

- Total Harmonic Distortion (THD):

$$k = \sqrt{\frac{\sum_{n=2} A_n^2}{A_1^2 + \sum_{n=2} A_n^2}} \quad k_v = \sqrt{\frac{A_v^2}{\sum_{n=1} A_n^2}} \quad v = 1, 2, \dots, N \quad k = \sqrt{k_2^2 + k_3^2 + \dots + k_N^2}$$

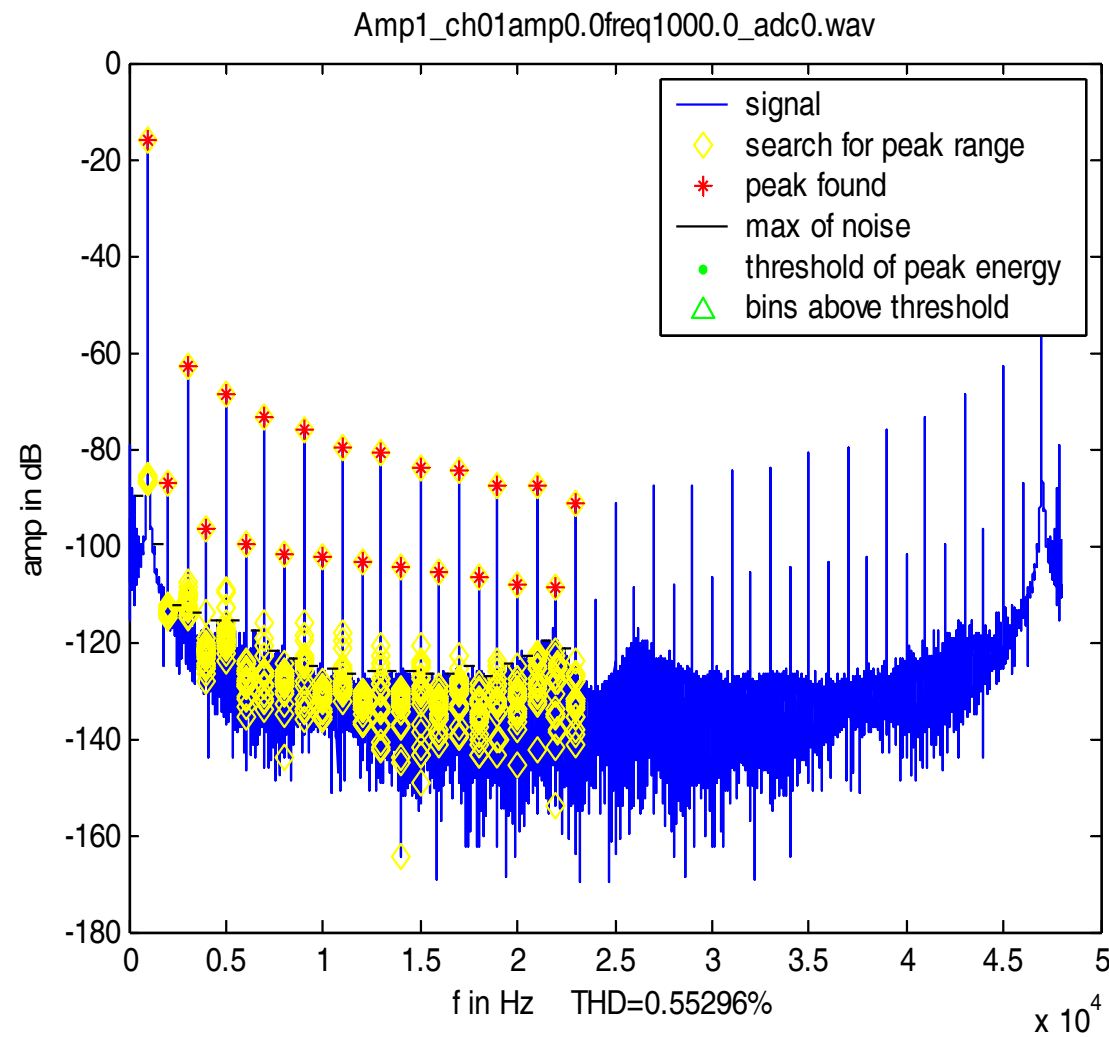
- Total Harmonic Distortion + Noise (THD+N):

$$THD + N = \sqrt{\frac{E_{out} - E_{in}}{E_{out}}} \quad X_{out}(f) = X_{in}(f) \cdot H_{notch}(f)$$

- Signal In Noise And Distortion (SINAD)
  - THD+N in dB

# THD

- Beispiel



## Weiterführende Literatur

- W. J. Rugh. **Nonlinear System Theory**. The Johns Hopkins University Press, Baltimore and London, 1981, ISBN: 0-8018-2549-0
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