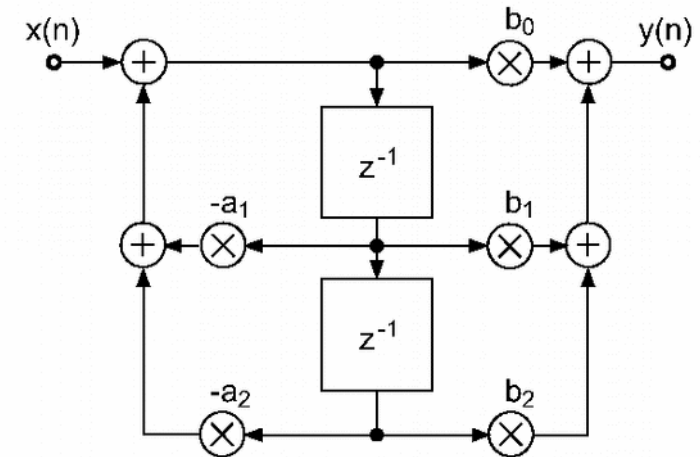
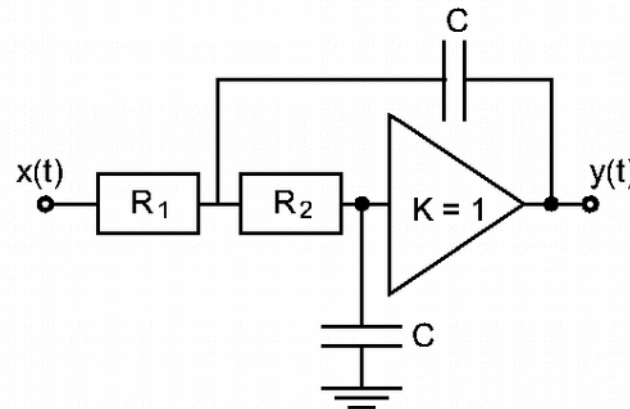
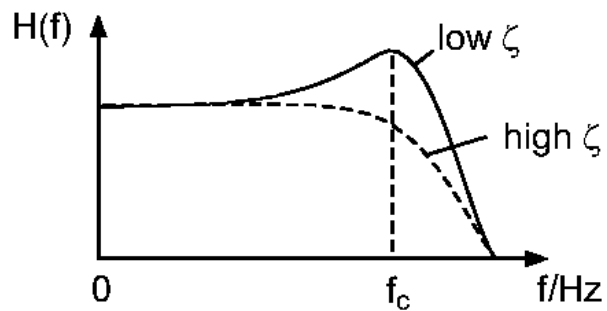


# Filter

- Ziele
- Parameter
- Entwurf



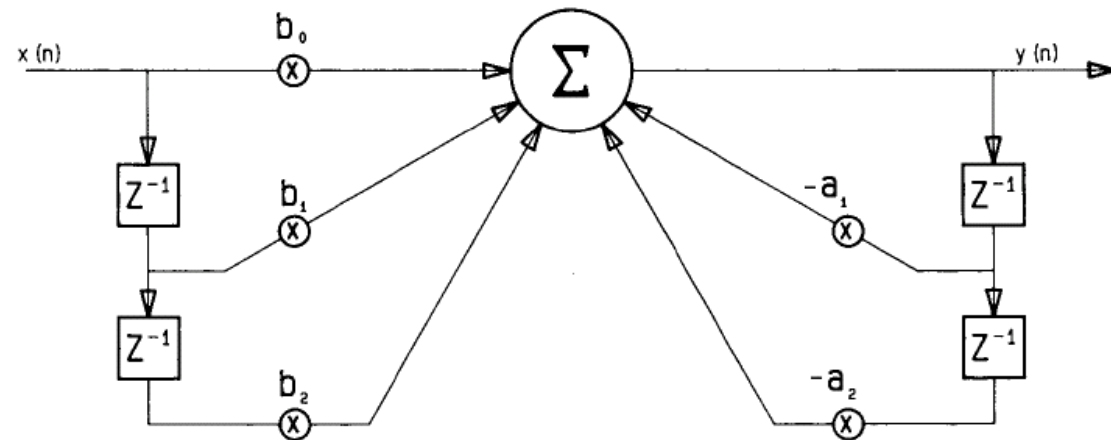
Zölzer (2002)

# Beschreibung

- Übertragungsfunktion  $H(z)$ ,  $H(\omega)$
- Impulsantwort  $h[n]$ :
  - Finite Impulse Response (FIR)
  - Infinite Impulse Response (IIR)
- Differenzengleichung  $y[n]$
- Pol-Nullstellen-Diagramm

# Biquad-Filter

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + \underbrace{a_1 z^{-1}}_{2r \cos \varphi} + \underbrace{a_2 z^{-2}}_{r^2}}$$



Datorro (1988)

- Stabil bei  $r < 1$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$$

- Beispiel: Tiefpass:

$$b_0 = 1 / (1 + 2 \cdot d \cdot F + F^2)$$

$$b_1 = 2b_0$$

$$b_2 = b_0$$

$$a_1 = 2b_0(1 - F^2)$$

$$a_2 = b_0(1 - 2 \cdot d \cdot F + F^2)$$

$$F = \frac{1}{\tan(\pi f_c / f_s)} \quad \varphi = 2\pi f_c / f_s$$

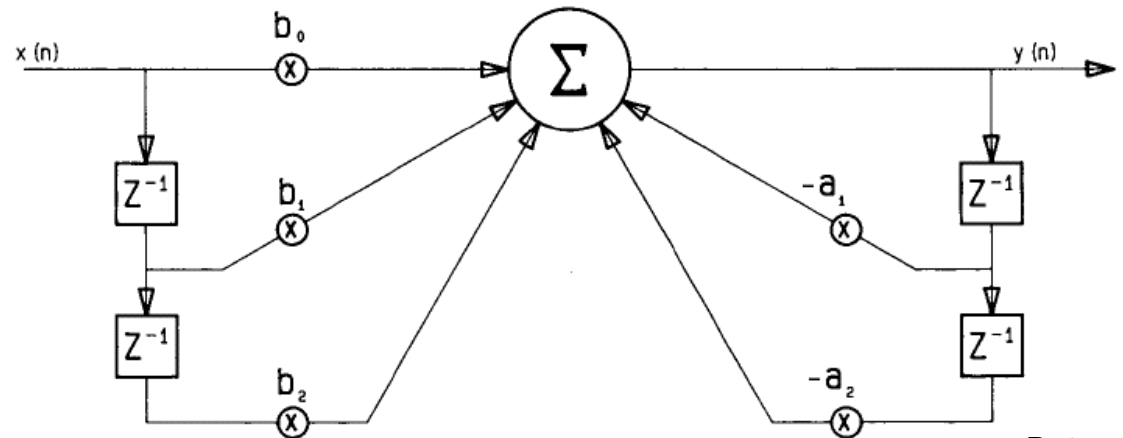
$f_c$  ... Analoge Grenzfrequenz

$f_s$  ... Abtastfrequenz (*sampling frequency*)

$d$  ... Dämpfungsfaktor

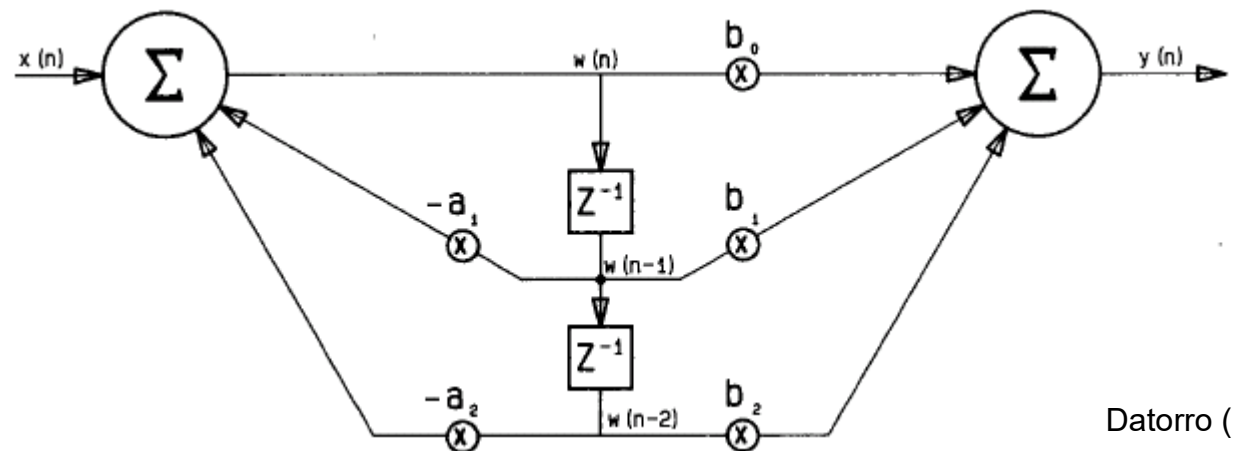
# Direktformen

- Direktform I:



Datorro (1988)

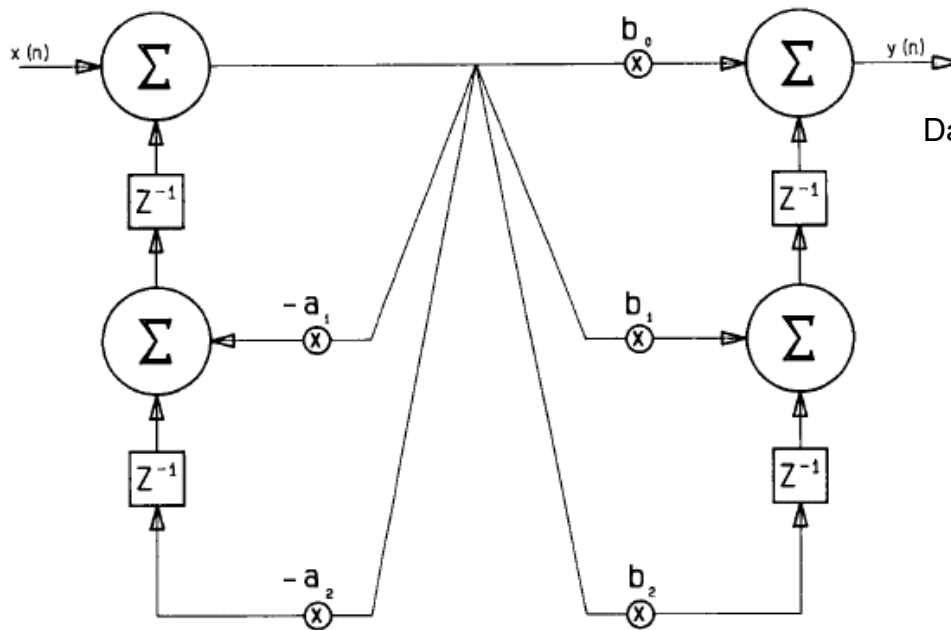
- Direktform II, kanonische Form:



Datorro (1988)

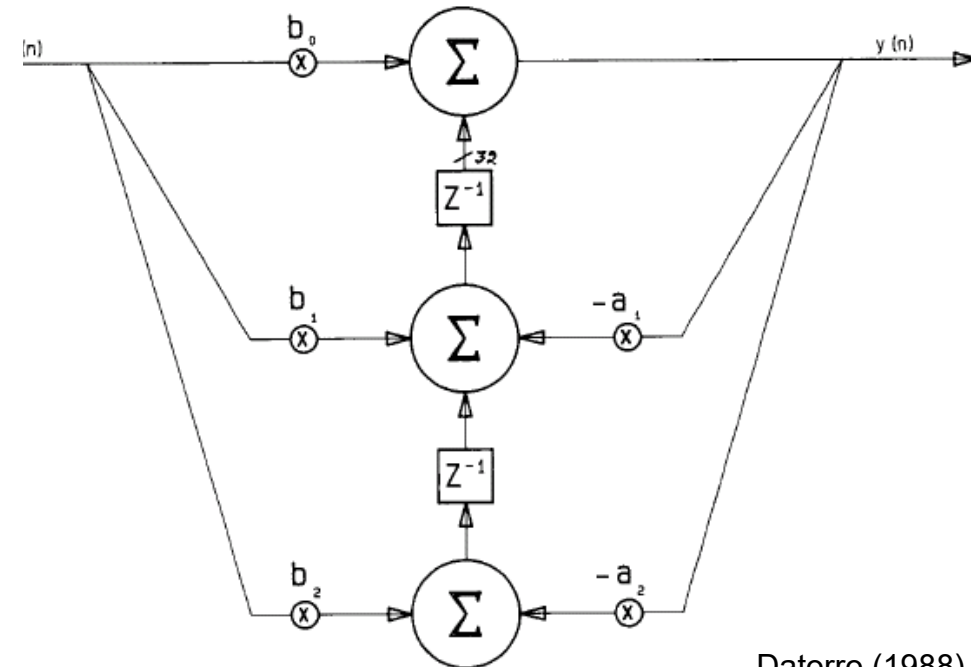
# Transponierte Direktformen

- Transp. Direktform I:



Datorro (1988)

- Transp. Direktform II:



Datorro (1988)

# Skalierung

- Rechenoperationen:
  - Multiplikation:  $16\text{bit} \times 16\text{bit} = 31\text{bit}$
  - Addition:  $16\text{bit} + 16\text{bit} = 17\text{bit}$
- Anforderungen:
  - Systemwortbreite: höher als Signalauflösung
  - Akkumulator: deutlich höhere Wortbreite
    - Signal: 16bit – System: 24bit – Akku: 48bit
  - Fließkommaprozessoren:
    - Signal: 24bit – System: 32 floating point
  - Skalierung des Signals, wenn notwendig!

# Filter höherer Ordnung

- Zerlegung der gesamten Filterstruktur in Biquad-Teile:
  - Serienschaltung
  - Parallelschaltung:
    - Unüblich: jeder Parameter kann das Verhalten des gesamten Filter verändern
- Unity gain filter:
  - Überlauf vermeiden an internen Knoten
  - Gefährliche Pole mit Nullstellen in einem Filter “aufheben”

# Überlauf

- Addition:
  - 1-er Komplement
    - Vorzeichen im MSB,
    - Einfachere Implementierung
  - 2-er Komplement für Addition
    - Wertetabelle:
    - Übliche Implementierung
    - Transparent für Überlauf:

Dezimal	Hex
32767	7FFF
1	1
0	0
-1	FFFF
-2	FFFE
-32768	8000

$$32500 + 512 = 33012 \dots 7EF4_H + 200_H = 80F4_H \dots -32524$$

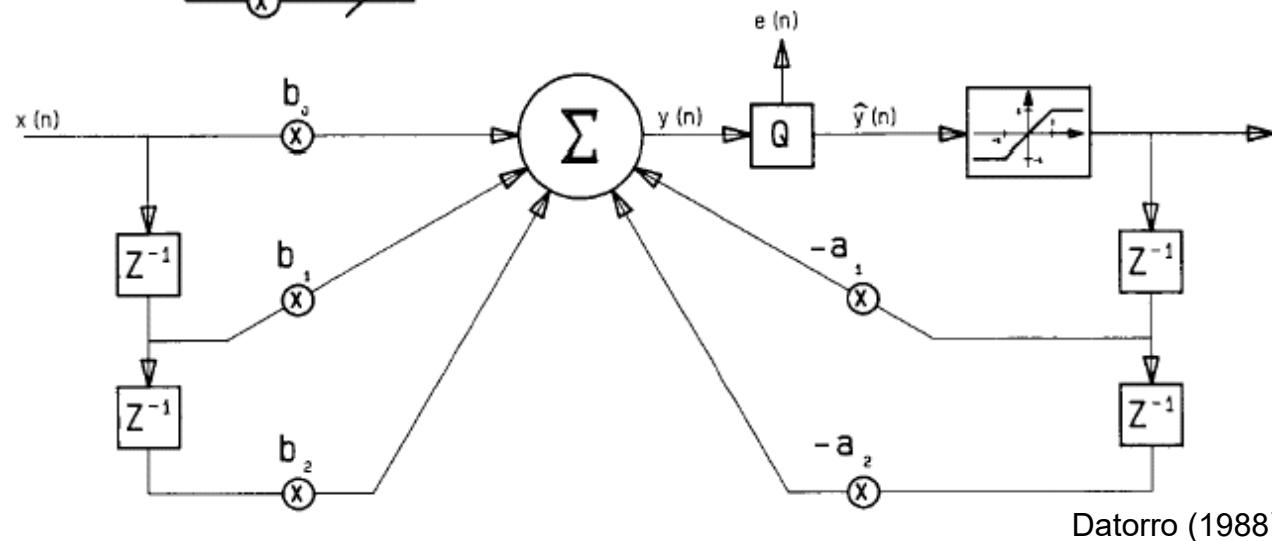
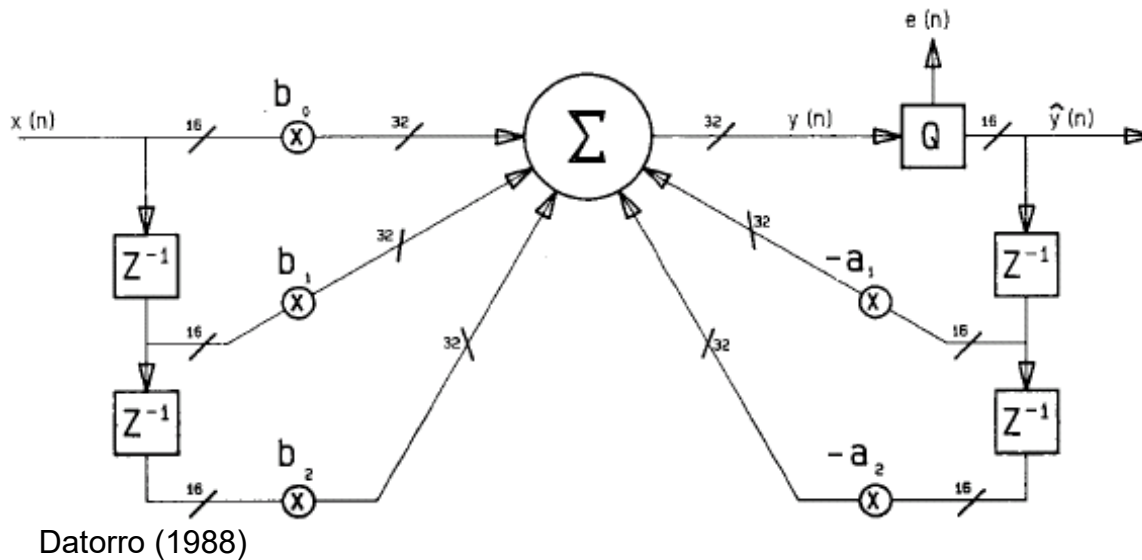
$$33012 + 488 = 33500 \dots 80F4_H + 1E8_H = 82DC_H \dots -32036$$

$$33500 - 1000 = 32500 \dots 82DC_H - 3E8_H = 7EF4_H \dots 32500$$



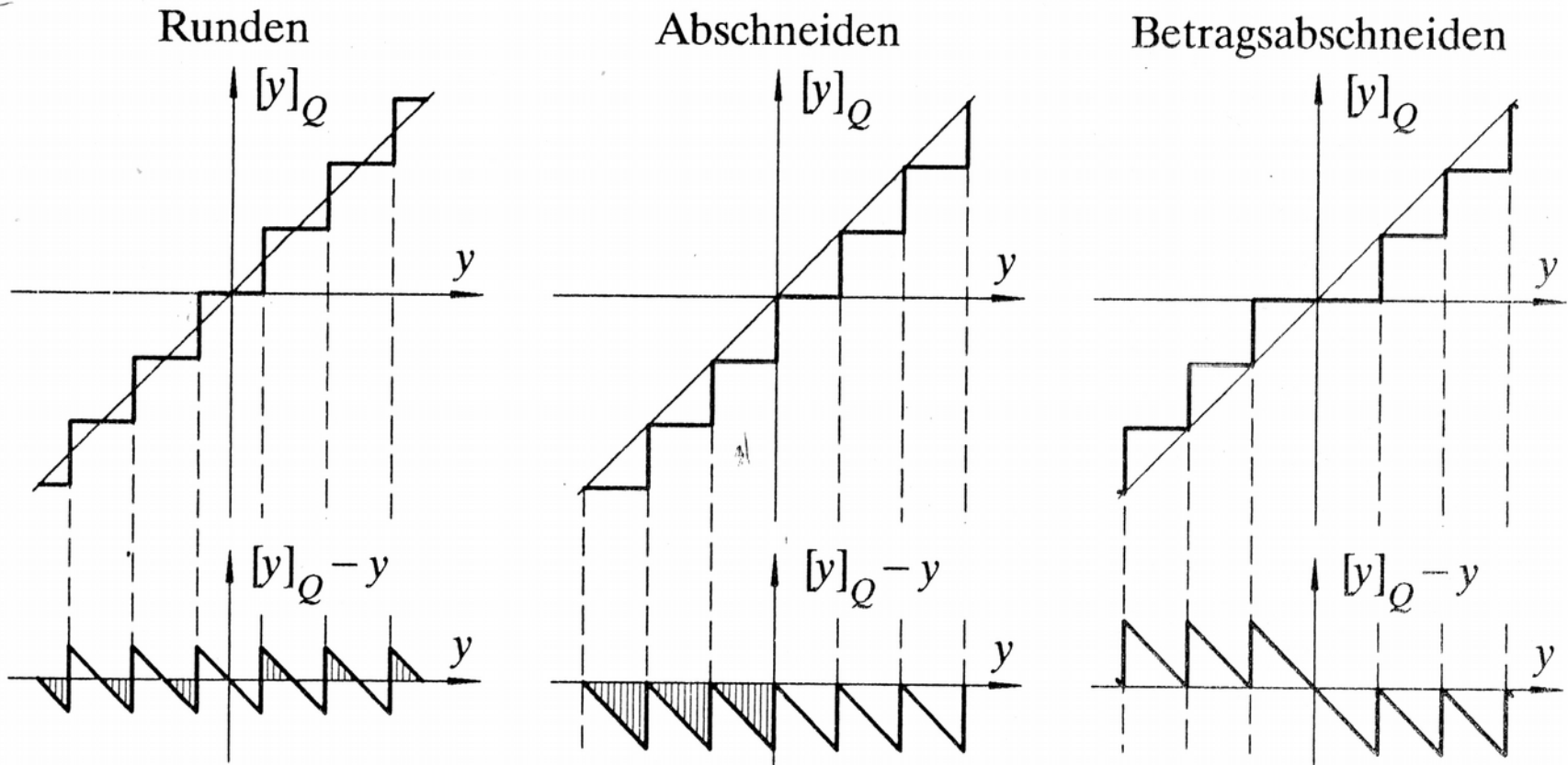
# Quantisierung des Signals

- Direktform I, Quantisierung nach dem Akku



# Runden oder Abschneiden

- Vergleich:

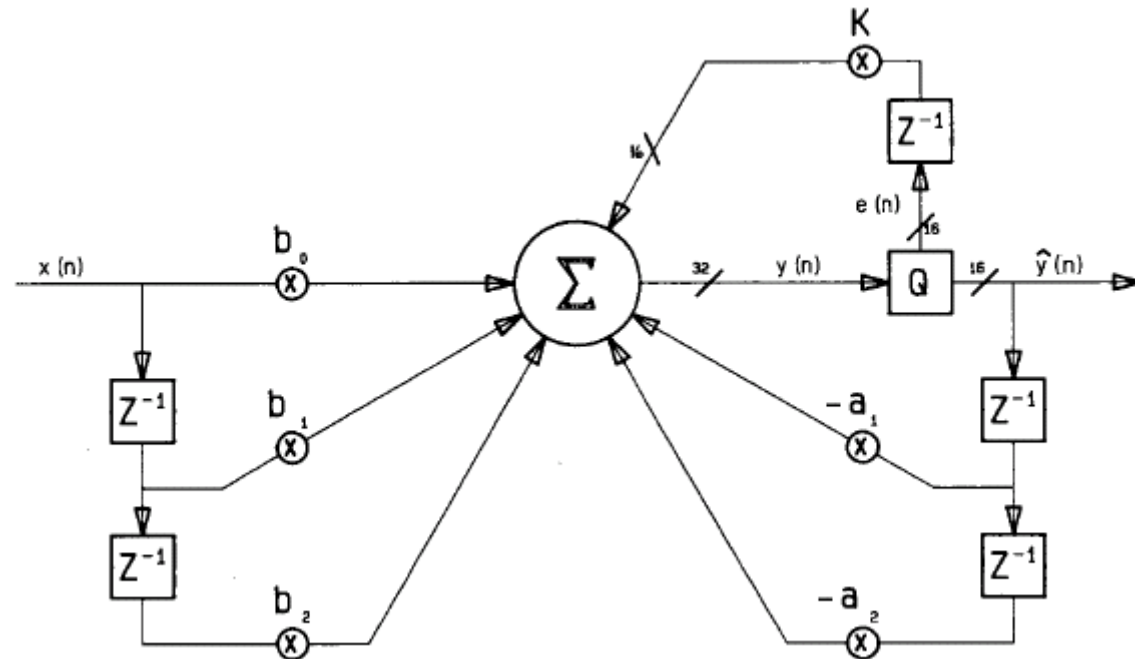


# Fehlerrückkopplung

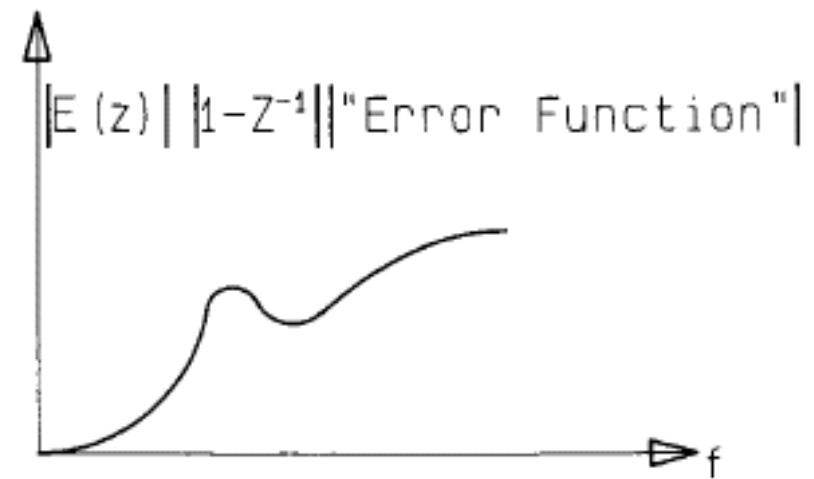
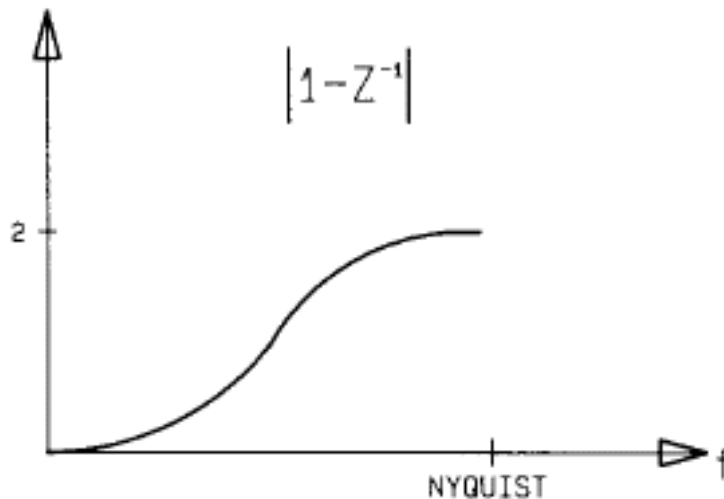
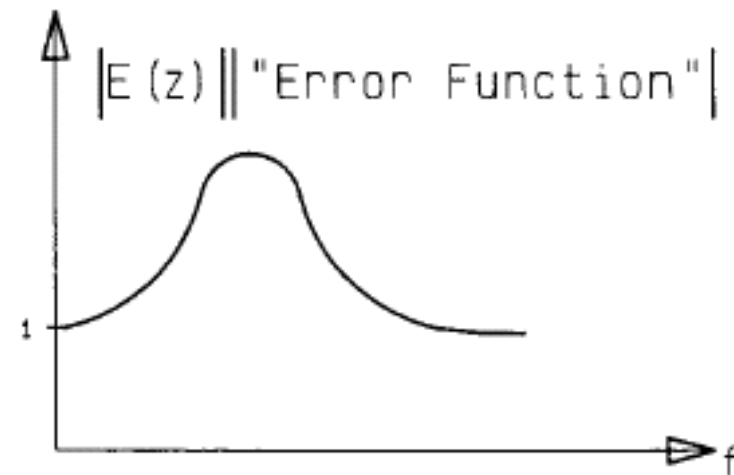
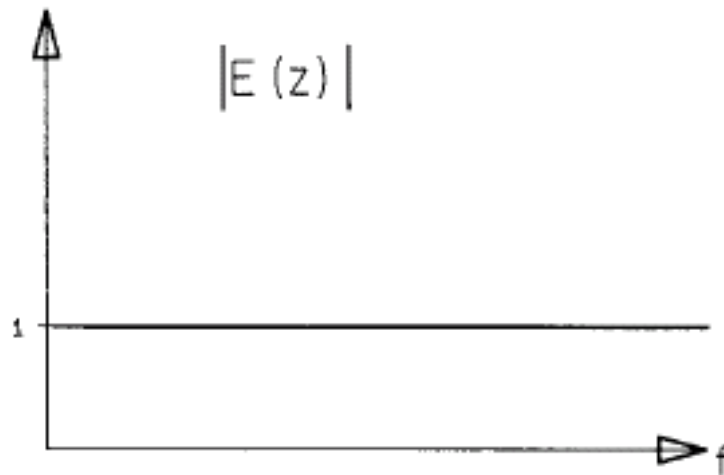
- Eigenschaften des Quantisierungsfehlers

$$\hat{Y}(Z) = X(Z) \cdot \frac{\sum b_i z^{-i}}{1 + \sum a_i z^{-i}} - E(Z) \cdot \frac{1}{1 + \sum a_i z^{-i}}$$

- Abhilfe: Rückkopplung



# Error Spectrum Shaping

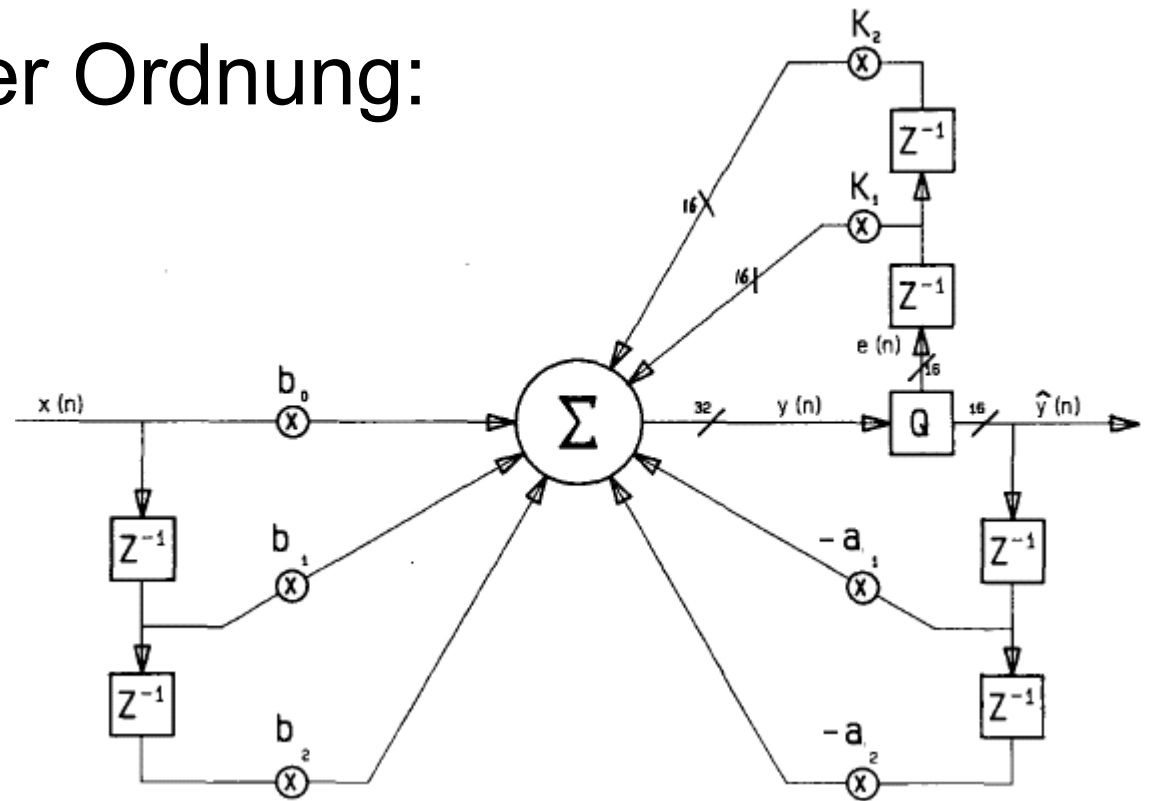


## Error Spectrum Shaping (2)

- Noise Shaping höherer Ordnung:

$K_1$	$K_2$	Region $\theta$
+2	-1	0 Twice
-2	-1	$\pi$ Twice
0	+1	0 and $\pi$
+1	-1	$\pi/3$ Twice
-1	-1	$2\pi/3$ Twice
+1	0	0 Once
-1	0	$\pi$ Once

Datorro (1988)

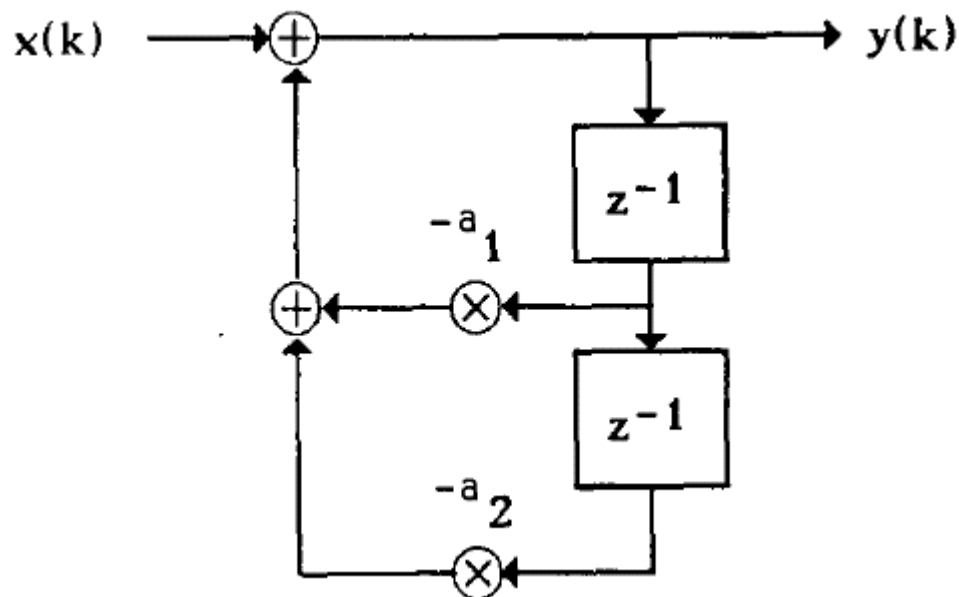


Datorro (1988)

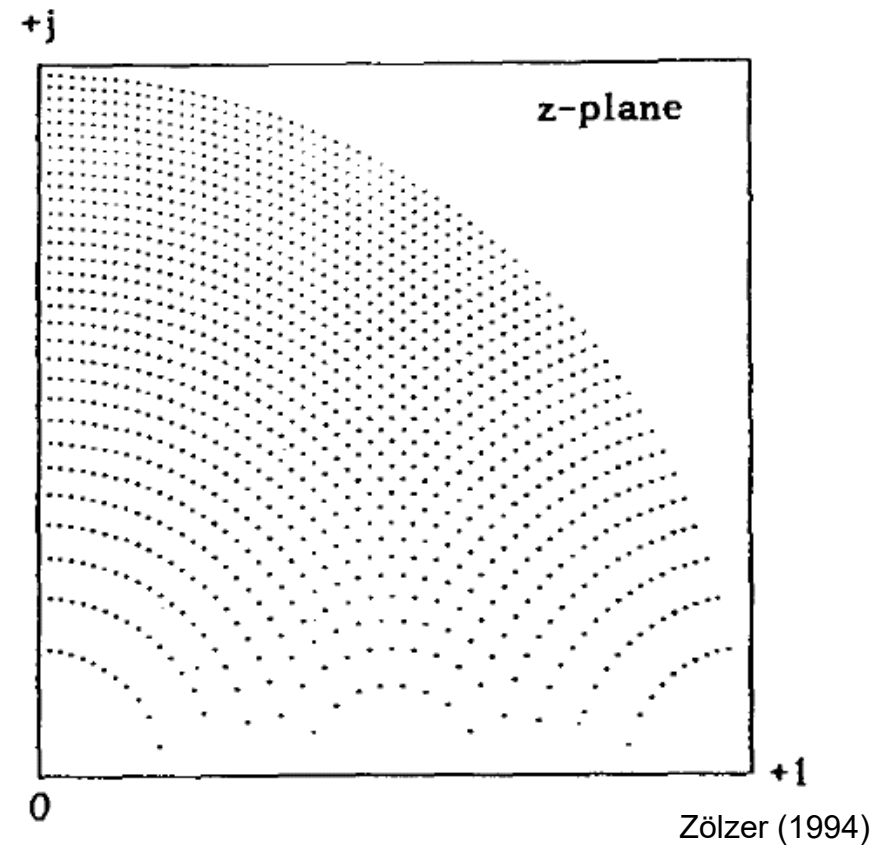
- Optimales Shaping?

# Quantisierung der Parameter

- Verschiebung der Pol-/Nullstellen



Zölzer (1994)

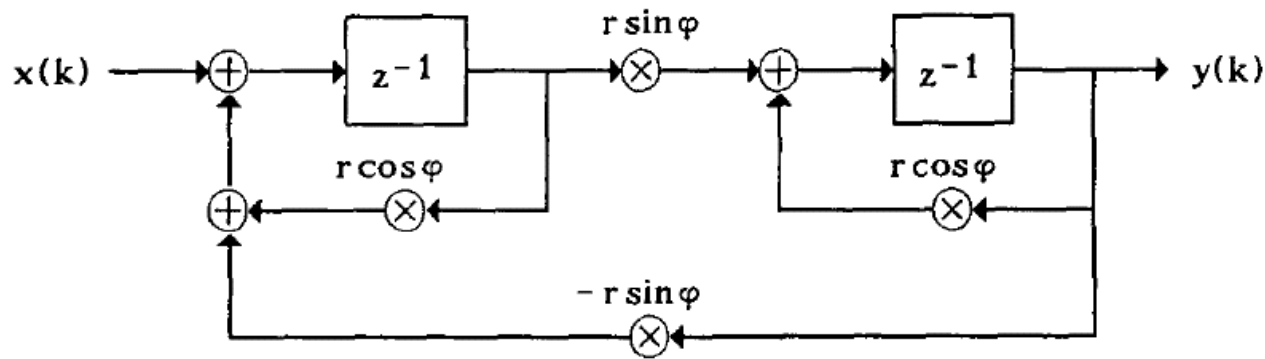


Zölzer (1994)

- Ungünstige Verteilung der Pol-/Nullstellen

# Gold-Rader-Struktur

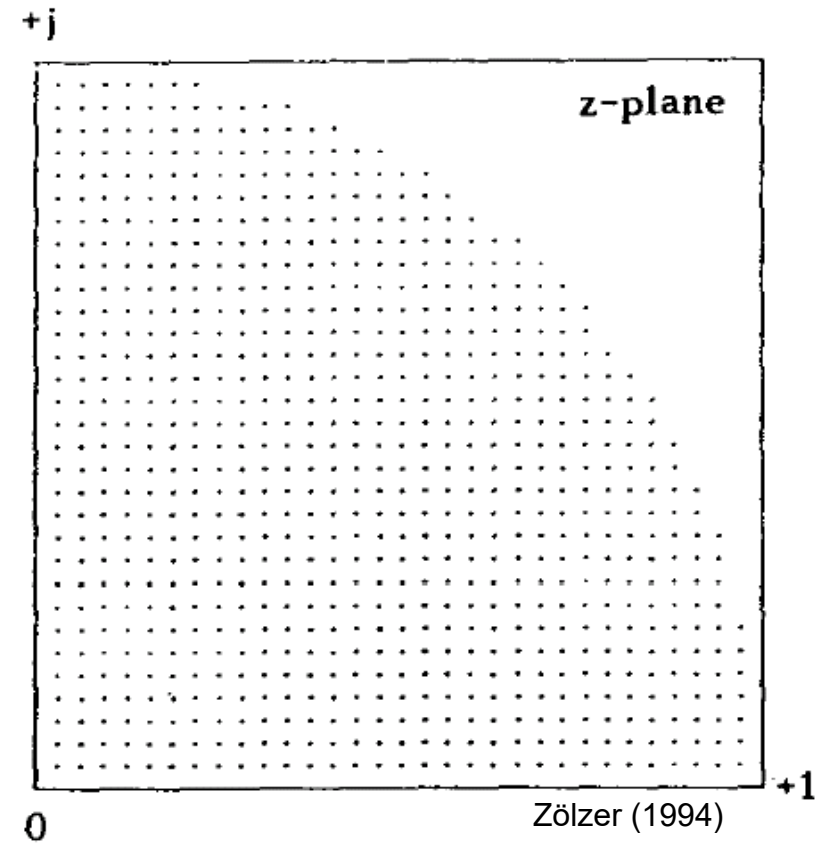
- Gleichmäßige Quantisierung der Z-Ebene



Zölzer (1994)

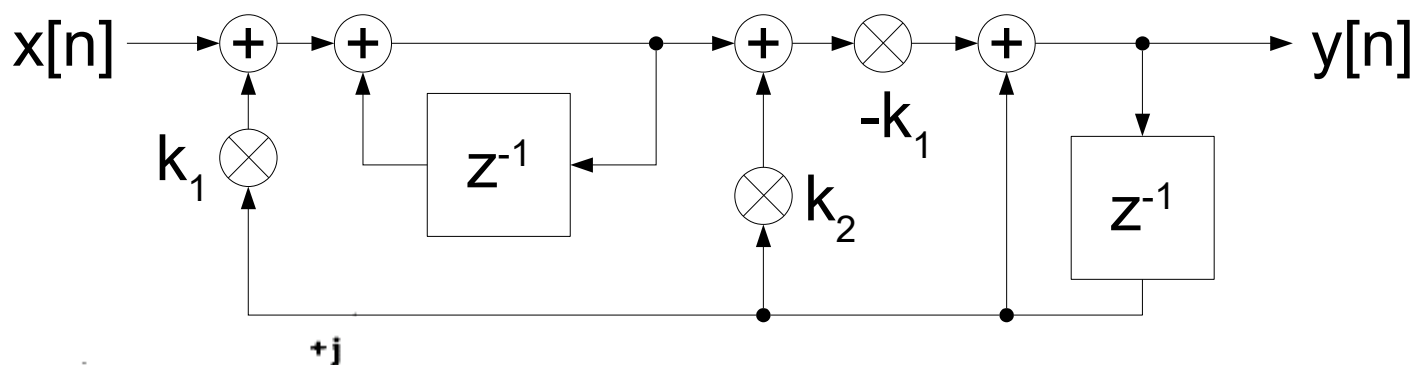
$$H(Z) = \frac{N(Z)}{1 + \underbrace{2 \Re\{z_\infty\}}_{a_1} \cdot z^{-1} + \underbrace{(\Re\{z_\infty\}^2 + \Im\{z_\infty\}^2)}_{a_2} \cdot z^{-2}}$$

$$z_\infty = r e^{j\varphi}$$

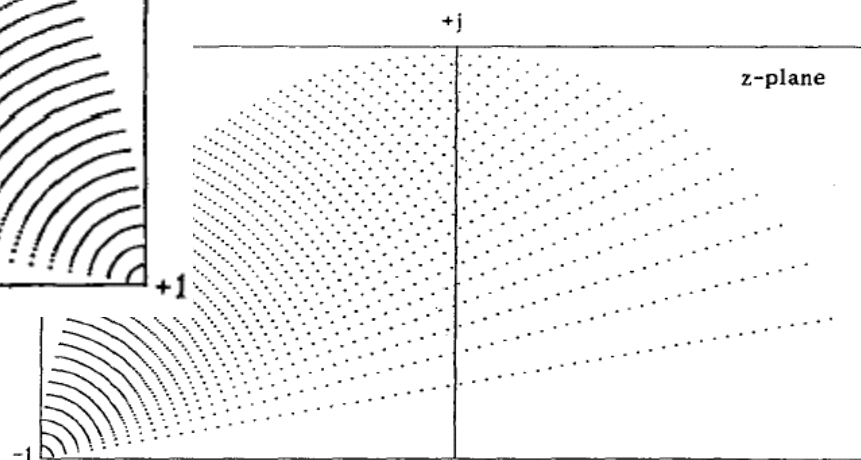
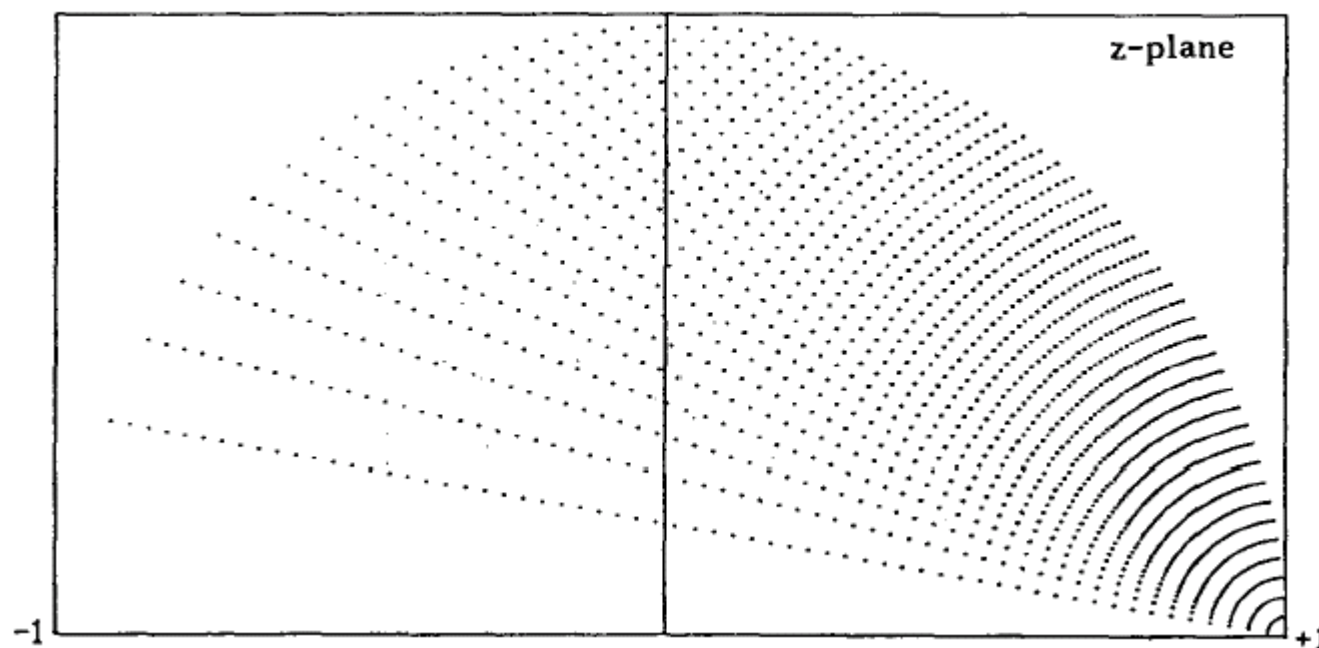


Zölzer (1994)

# Kingsbury-Struktur



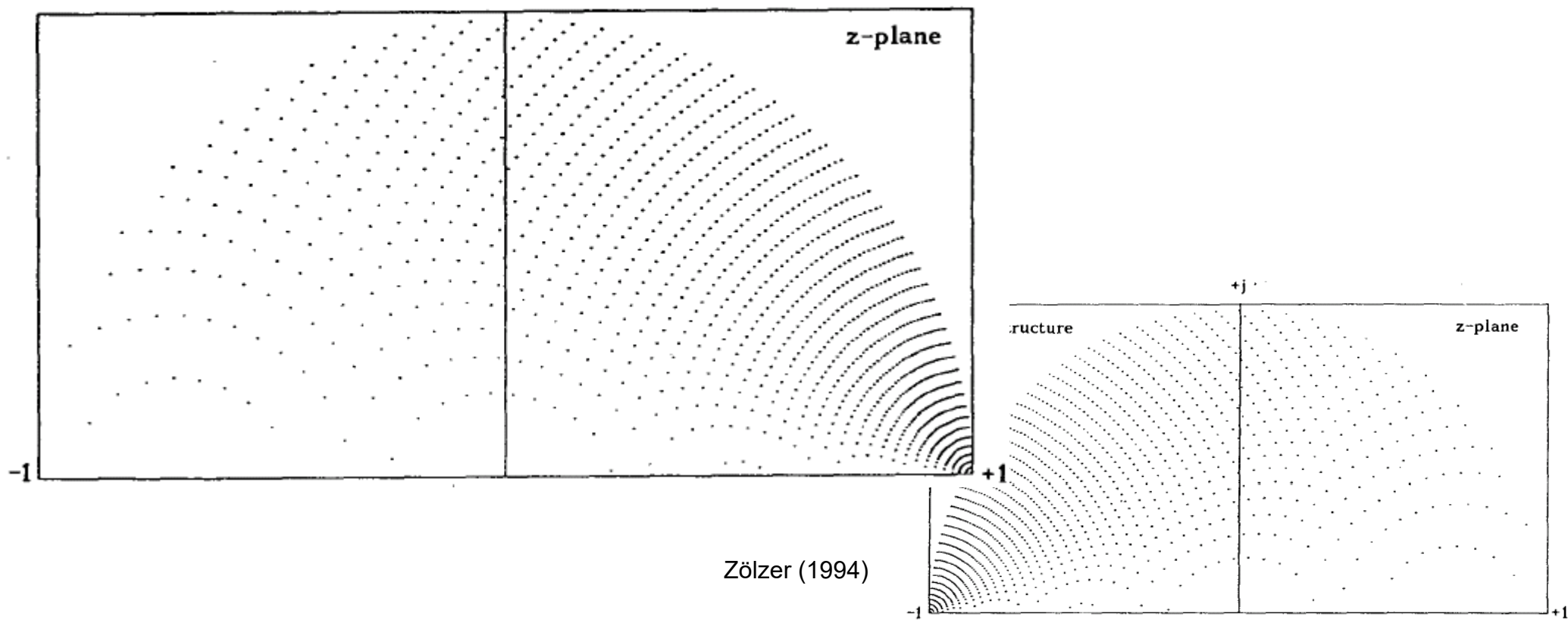
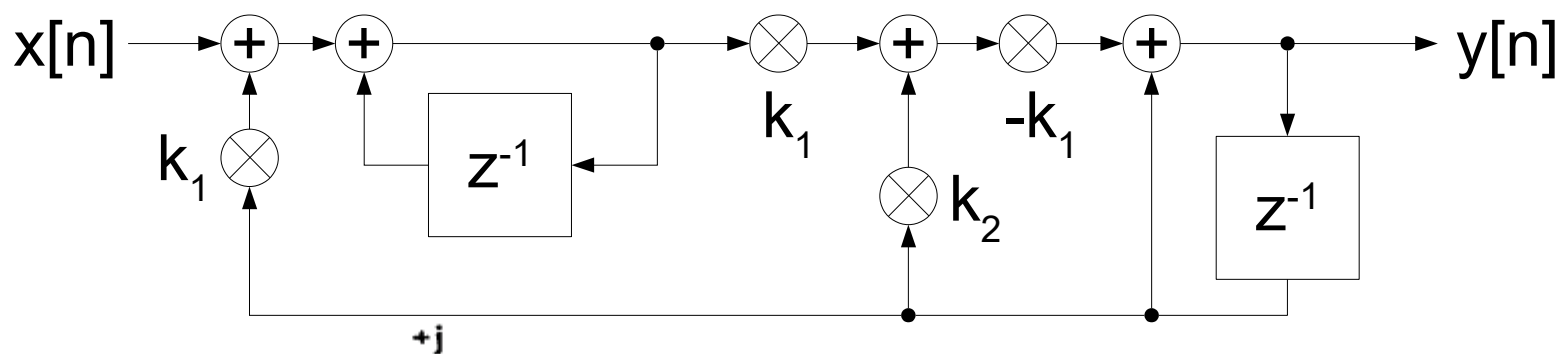
+j



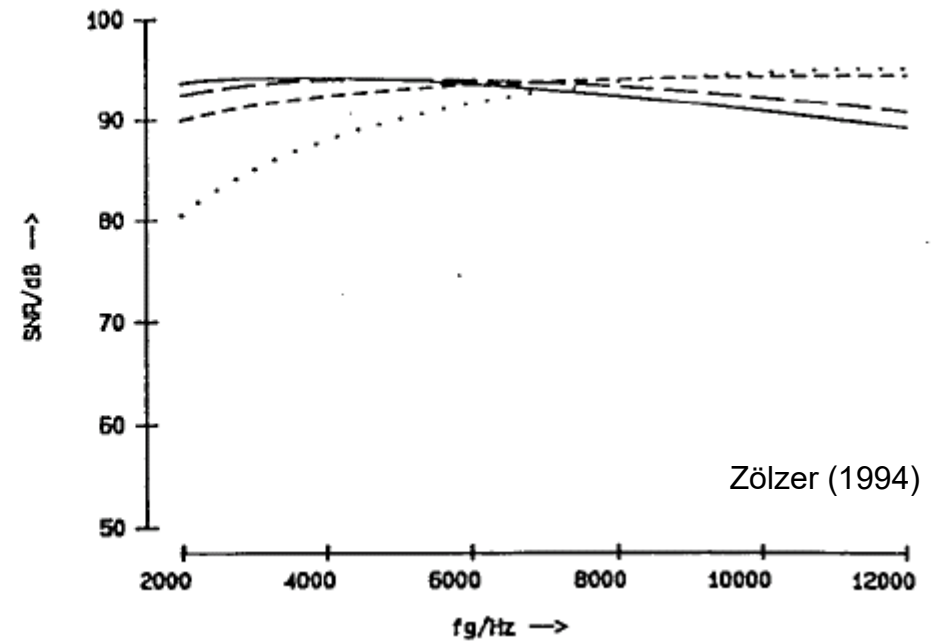
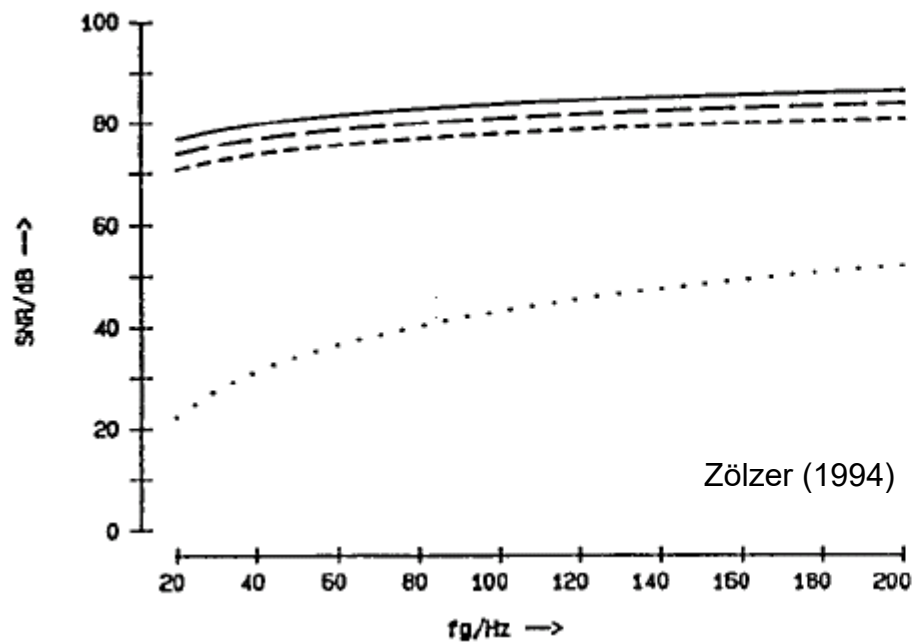
Zölzer (1994)



# Zölzer-Struktur



# Vergleich der Strukturen



- ..... Direktform
- - - Gold-Rader-Struktur
- - - Kingsbury-Struktur
- Zölzer-Struktur

# Parametrische Filter

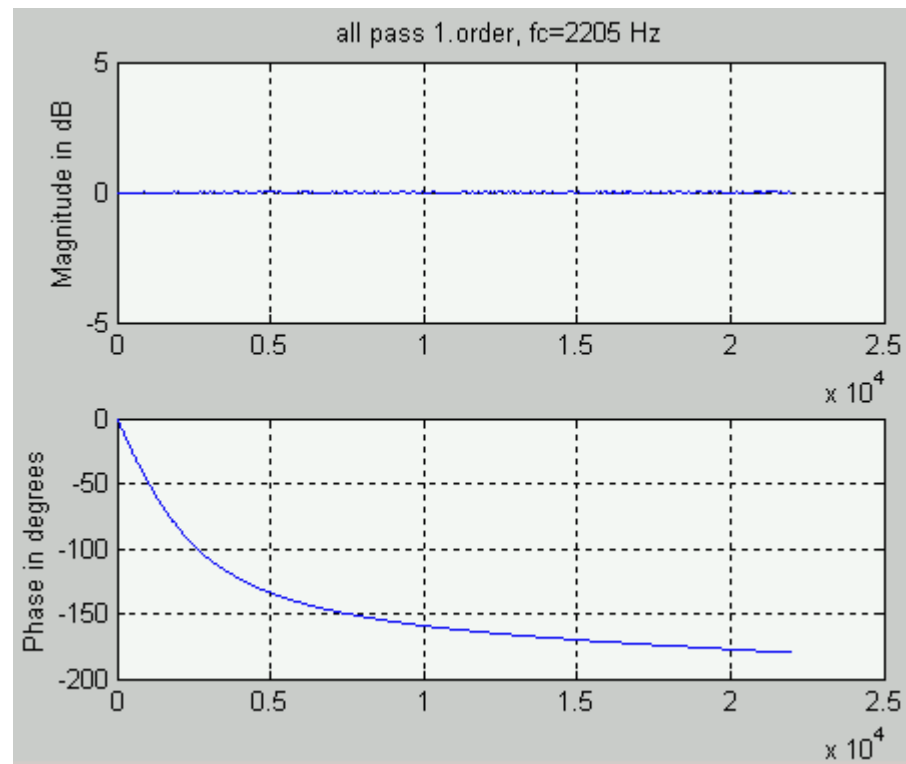
- Nachteile der Biquad-Struktur:
  - Charakteristik von allen Koeffizienten abhängig
  - Nur für statische Filter geeignet
- Ziele:
  - Unabhängige Koeffizienten
  - Stabil im jeweiligen Bereich
- Lösungen:
  - Spezielle Strukturen
  - Allpassfilter

# Allpassfilter

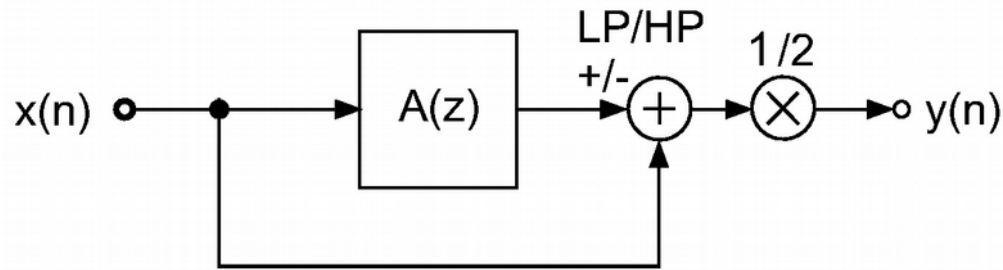
- Übertragungsfunktion:

$$A_1(z) = \frac{z^{-1} + c}{1 + c \cdot z^{-1}} \quad c = \frac{\tan(\pi f_c / f_s) - 1}{\tan(\pi f_c / f_s) + 1}$$

- Spektrum:



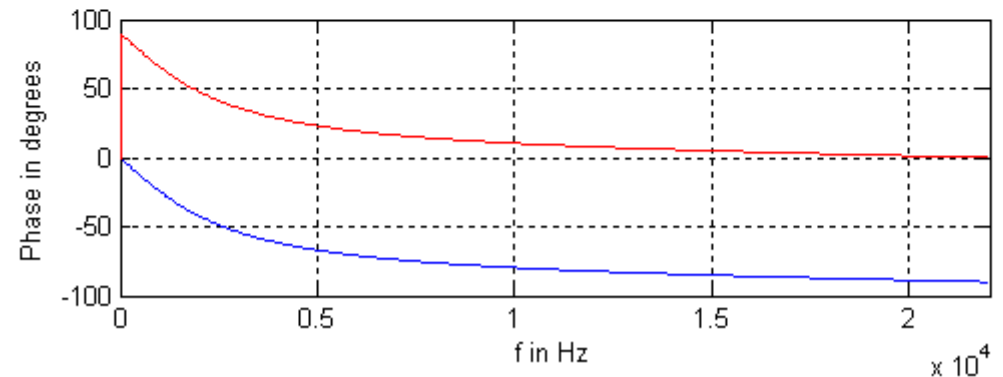
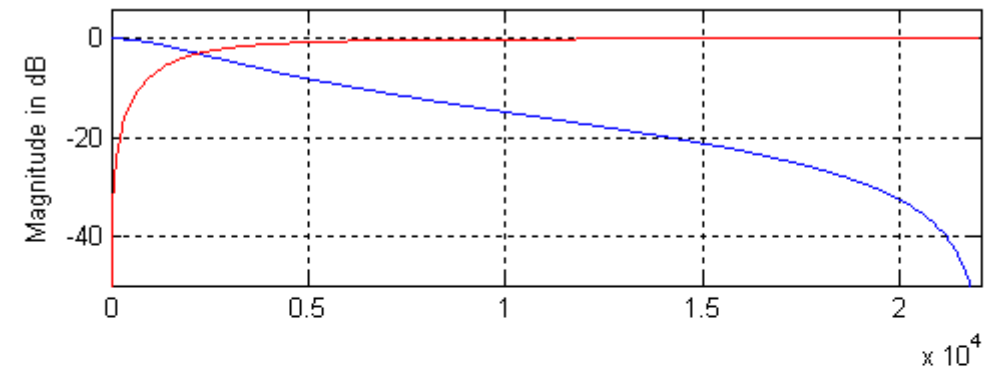
# Hoch-/Tiefpassfilter



Zölzer (2002)

$$H(z) = \frac{1}{2}(1 + A(z))$$

$$H(z) = \frac{1}{2}(1 - A(z))$$



# Allpassfilter 2-ter Ordnung

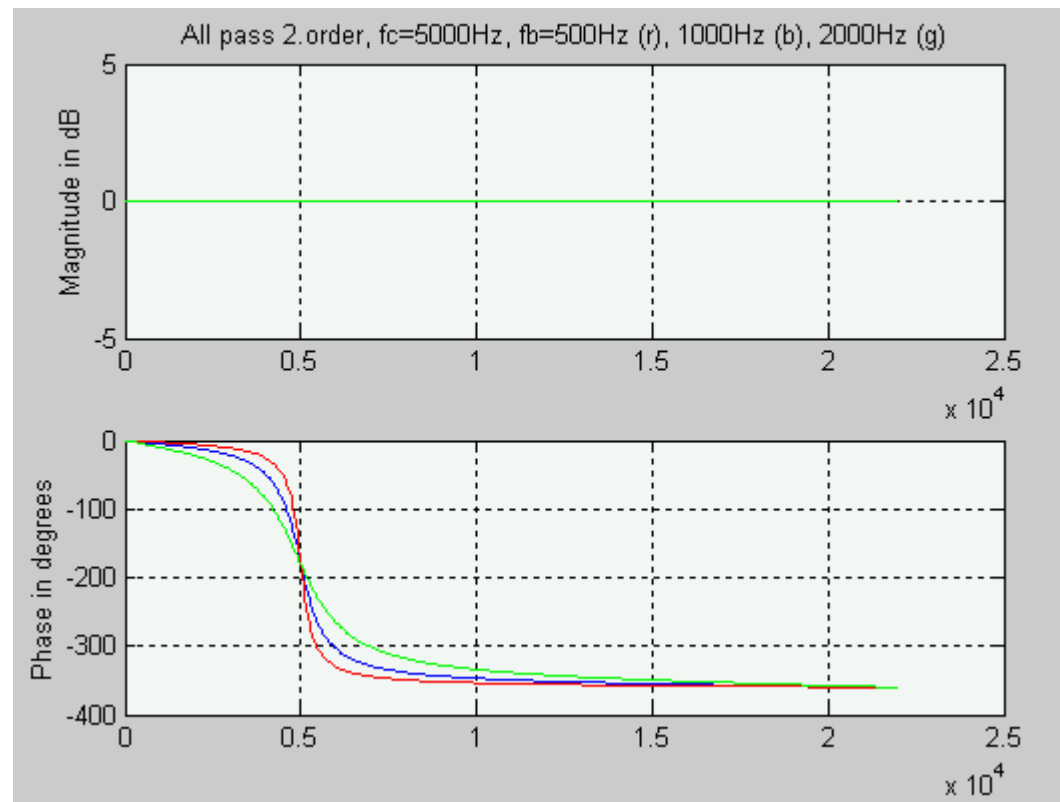
- Übertragungsfunktion:

$$b = \frac{\tan(\pi f_b / f_s) - 1}{\tan(\pi f_b / f_s) + 1}$$

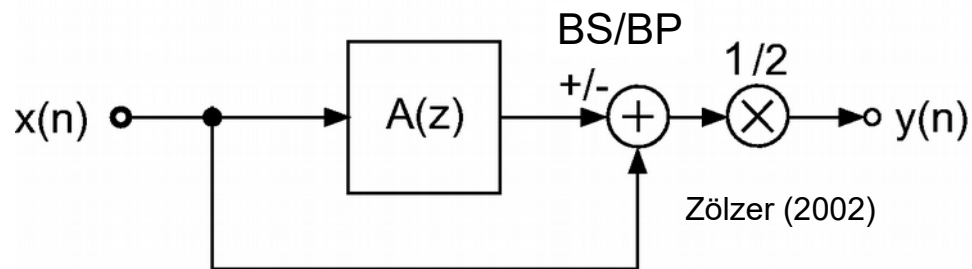
$$c = -\cos(2\pi f_c / f_s)$$

$$A_2(z) = \frac{z^{-2} + c(1-b)z^{-1} - b}{1 + c(1-b)z^{-1} - bz^{-2}}$$

- Spektrum:

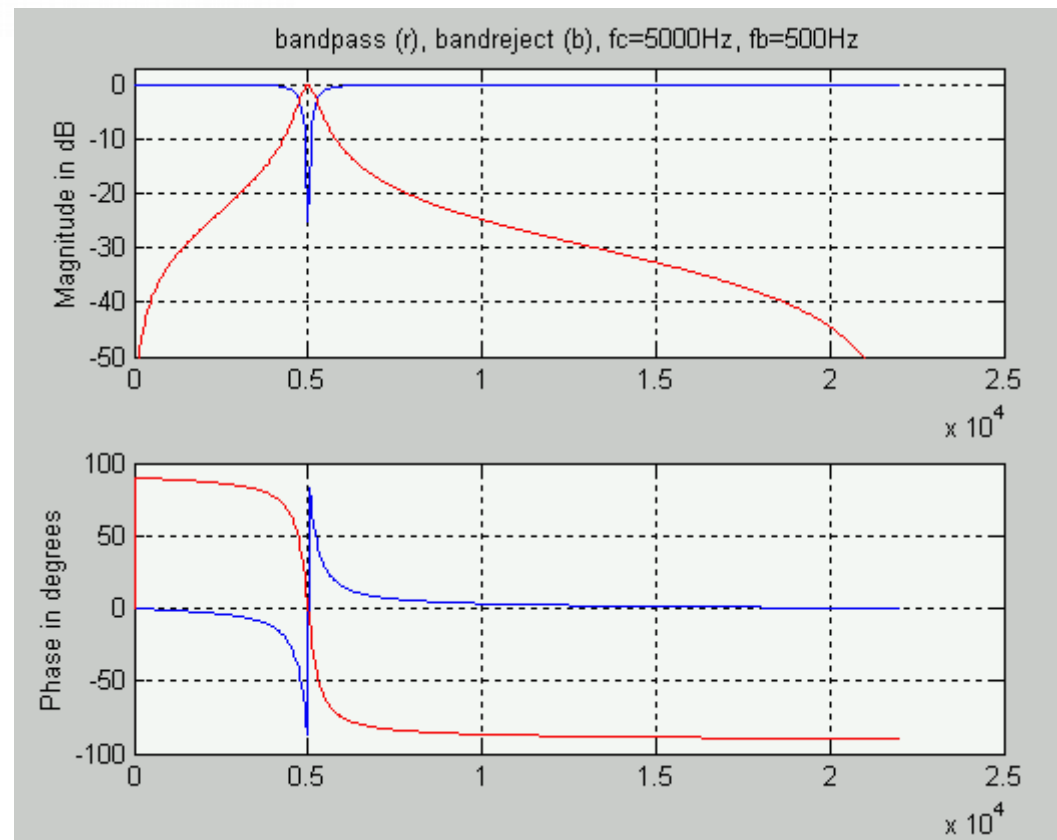


# Bandpass / Bandsperre



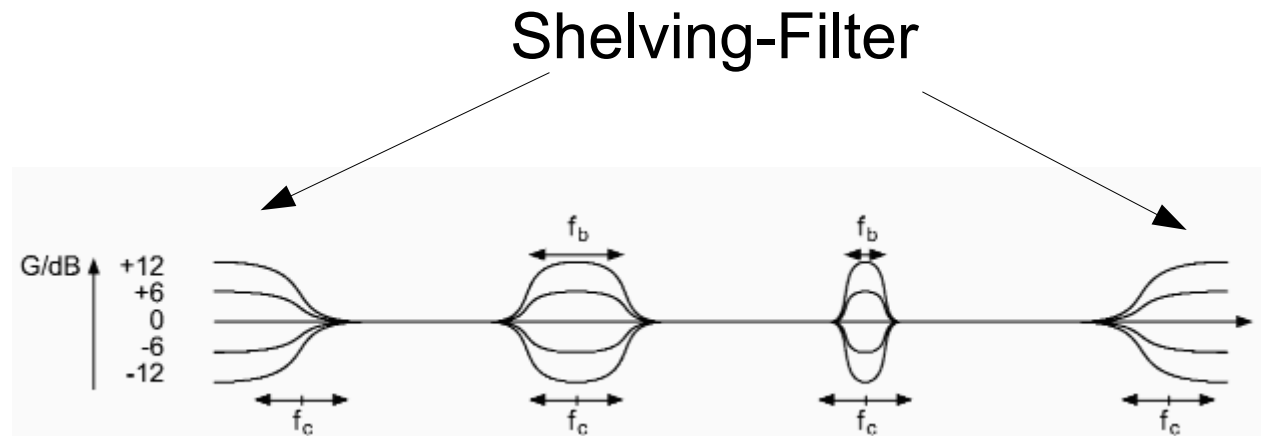
$$H(z) = \frac{1}{2} (1 + A_2(z))$$

$$H(z) = \frac{1}{2} (1 - A_2(z))$$



# Equalizer

- Parameter:



Zölzer (2002)

Peak / Notch-Filter



# Shelving Filter

- Kombination aus Tief- und Hochpassfilter
- Idee: Interessanter Bereich wird betont
- Hochpass:

$$\begin{array}{l} TP: \quad \frac{1}{2}[1 + A(z)] \\ HP: \quad G_\infty \cdot \frac{1}{2}[1 - A(z)] \end{array} \Bigg| +$$

---


$$H(z) = \frac{1}{2}[G_\infty + 1 + (1 - G_\infty)A(z)]$$

mit  $H_0 = G_\infty - 1 \rightarrow G_\infty = H_0 + 1$ :

$$H(z) = 1 + \frac{H_0}{2} \cdot [1 - A(z)]$$

Tiefpass:

$$\begin{array}{l} TP: \quad \frac{G_0}{2}[1 + A(z)] \\ HP: \quad \frac{1}{2}[1 - A(z)] \end{array} \Bigg| +$$

---


$$H(z) = \frac{1}{2}[G_0 + 1 + (G_0 - 1)A(z)]$$

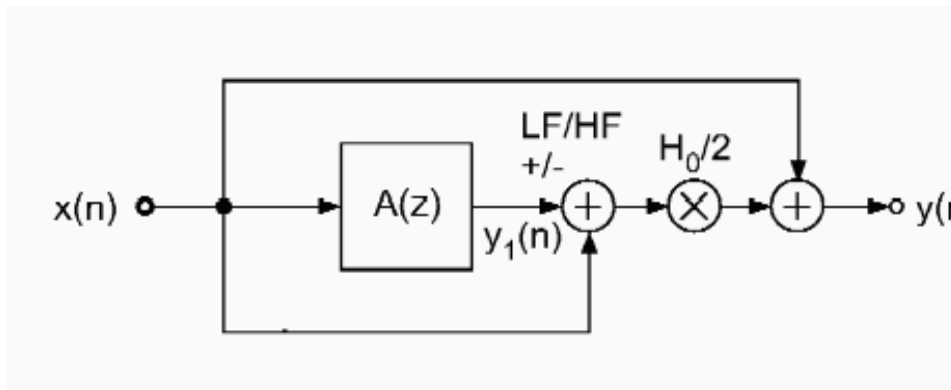
mit  $H_0 = G_0 - 1 \rightarrow G_0 = H_0 + 1$

$$H(z) = 1 + \frac{H_0}{2} \cdot [1 + A(z)]$$

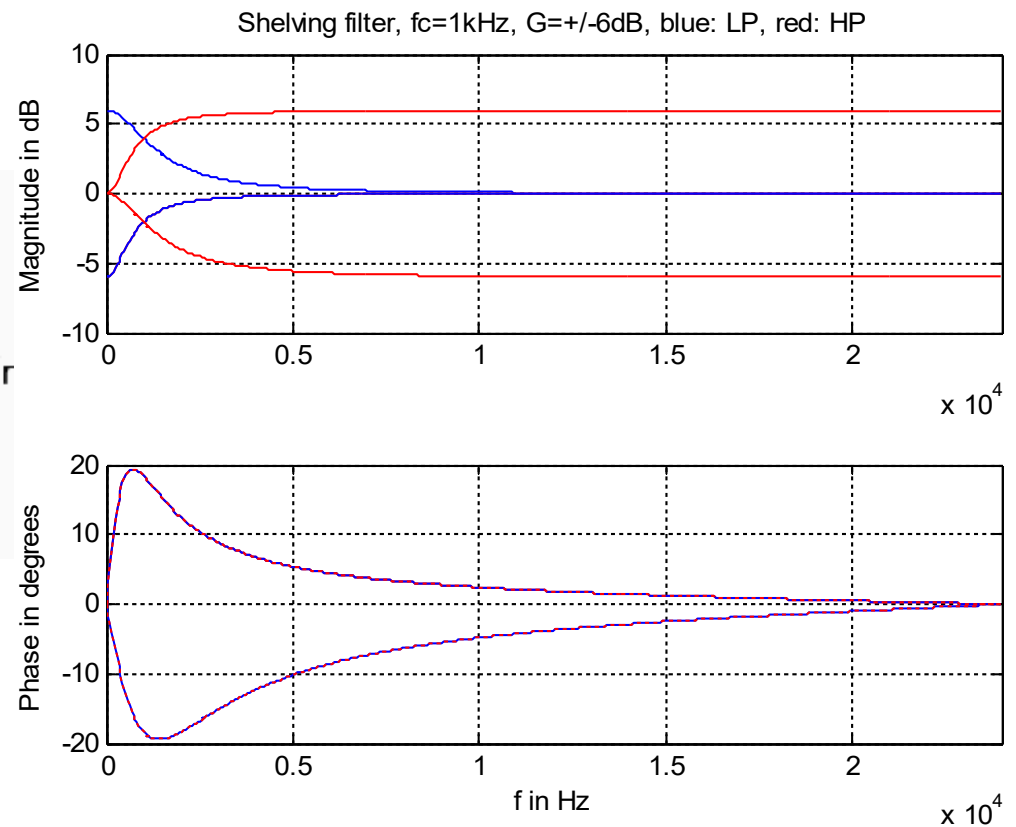
# Shelving Filter

$$H(z) = 1 + \frac{H_0}{2} \cdot [1 \pm A(z)]$$

$$A_1(z) = \frac{z^{-1} + c}{1 + c \cdot z^{-1}} \quad c = \frac{\tan(\pi f_c / f_s) - 1}{\tan(\pi f_c / f_s) + 1}$$



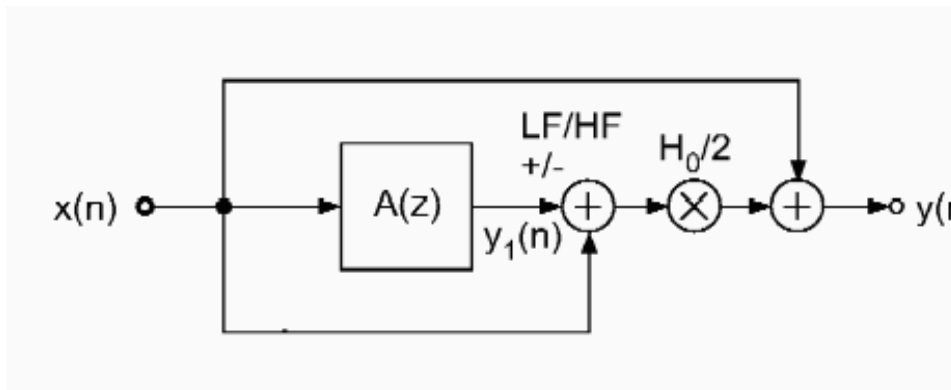
Zölzer (2002)



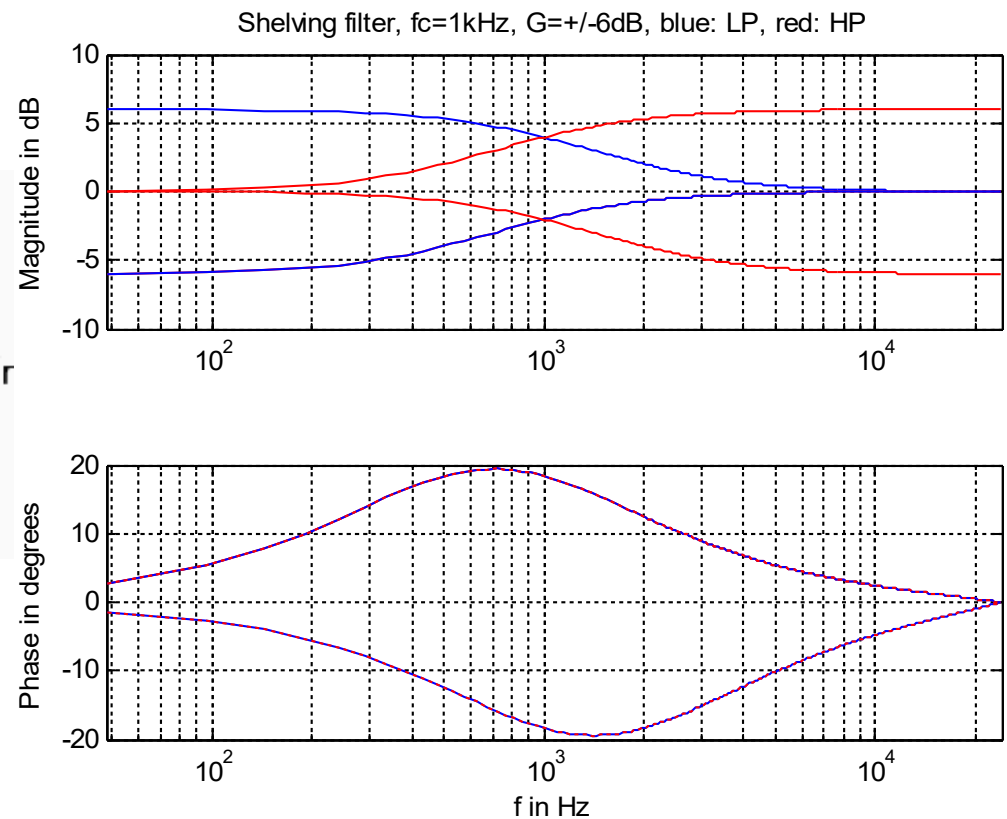
# Shelving Filter

$$H(z) = 1 + \frac{H_0}{2} \cdot [1 \pm A(z)]$$

$$A_1(z) = \frac{z^{-1} + c}{1 + c \cdot z^{-1}} \quad c = \frac{\tan(\pi f_c / f_s) - 1}{\tan(\pi f_c / f_s) + 1}$$



Zölzer (2002)



# Peakfilter

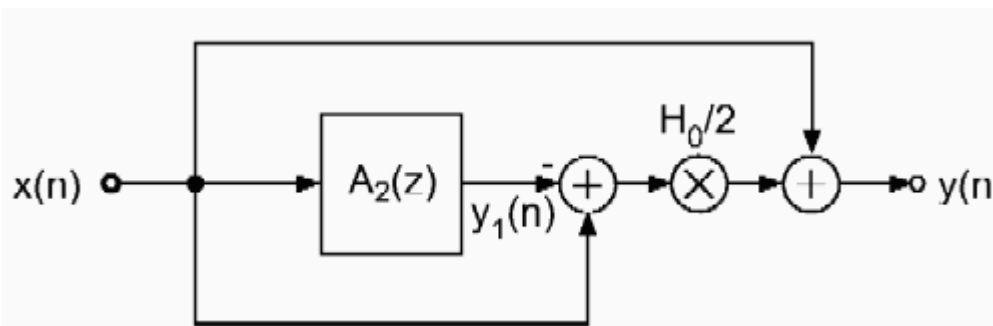
- Kombination aus Bandpass und Bandsperre

$$H(z) = 1 + \frac{H_0}{2} \cdot [1 - A_2(z)]$$

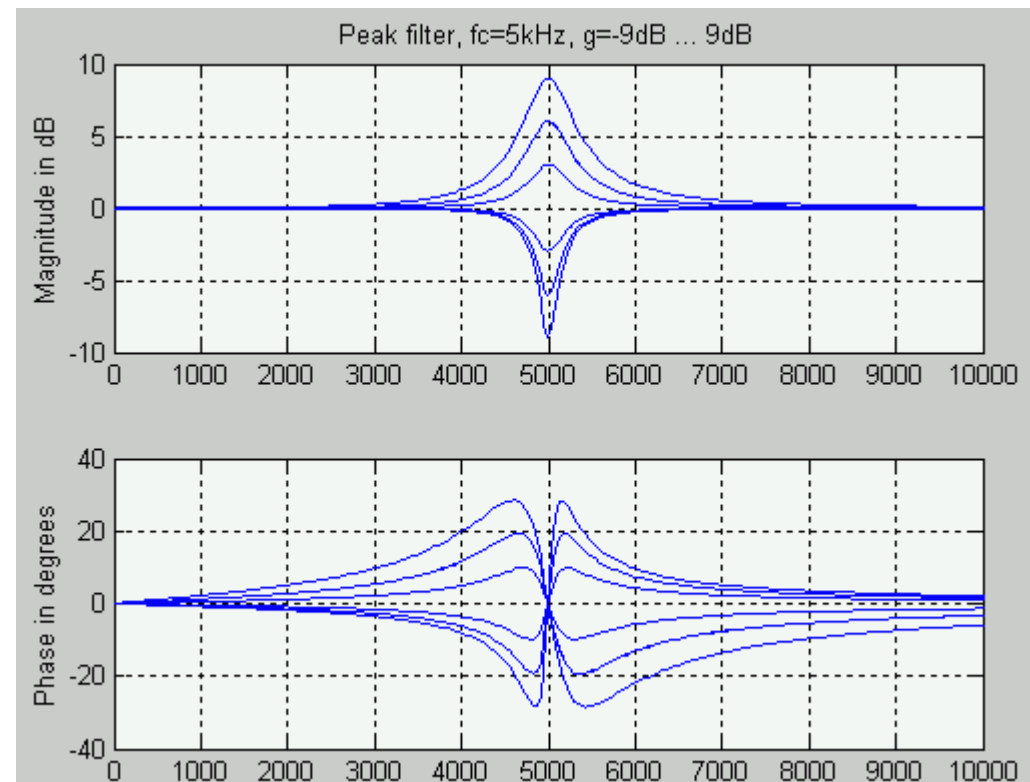
$$A_2(z) = \frac{z^{-2} + c(1-b)z^{-1} - b}{1 + c(1-b)z^{-1} - bz^{-2}}$$

$$b = \frac{\tan(\pi f_b / f_s) - 1}{\tan(\pi f_b / f_s) + 1}$$

$$c = -\cos(2\pi f_c / f_s)$$



Zölzer (2002)



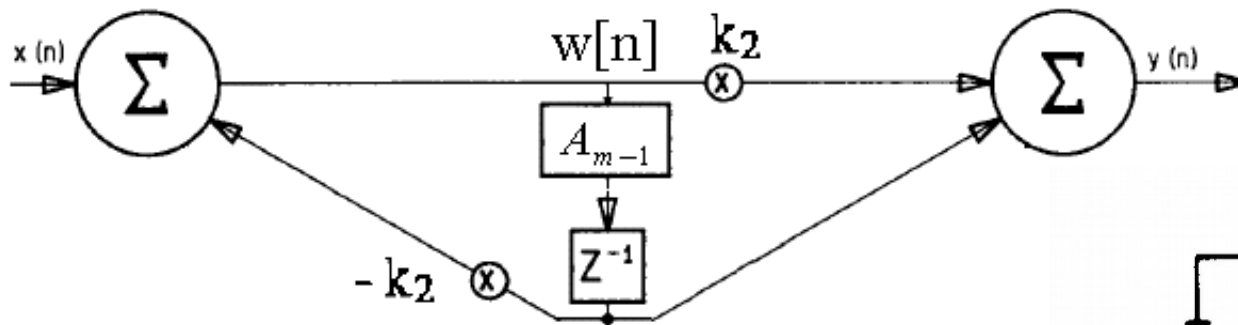
# Lattice Filter

- Problem: Parameter im Allpass nicht unabhängig:

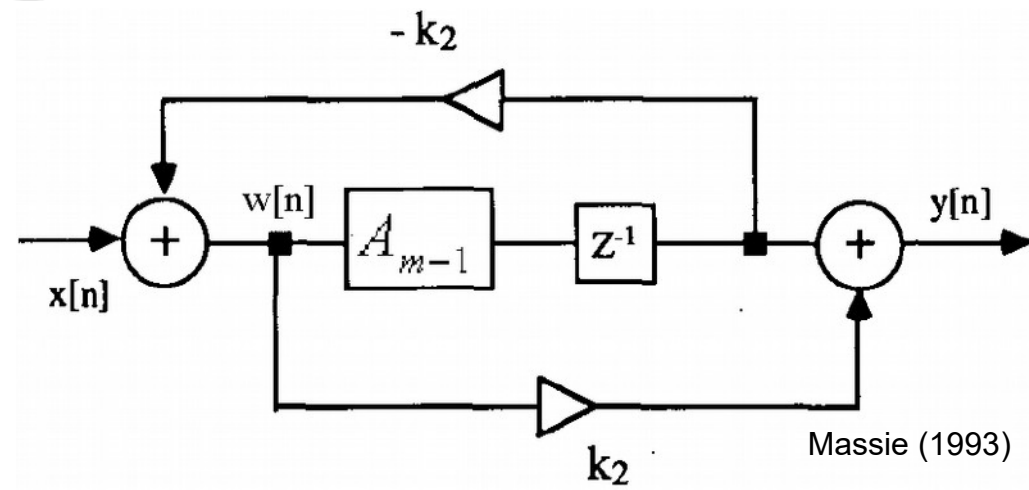
$$A_2(z) = \frac{z^{-2} + k_1(1 - k_2)z^{-1} - k_2}{1 + k_1(1 - k_2)z^{-1} - k_2z^{-2}}$$

- Rekursive Form:

$$A_m(z) = \frac{k_2 + z^{-1} \cdot A_{m-1}(Z)}{1 + k_2 z^{-1} \cdot A_{m-1}(Z)}$$

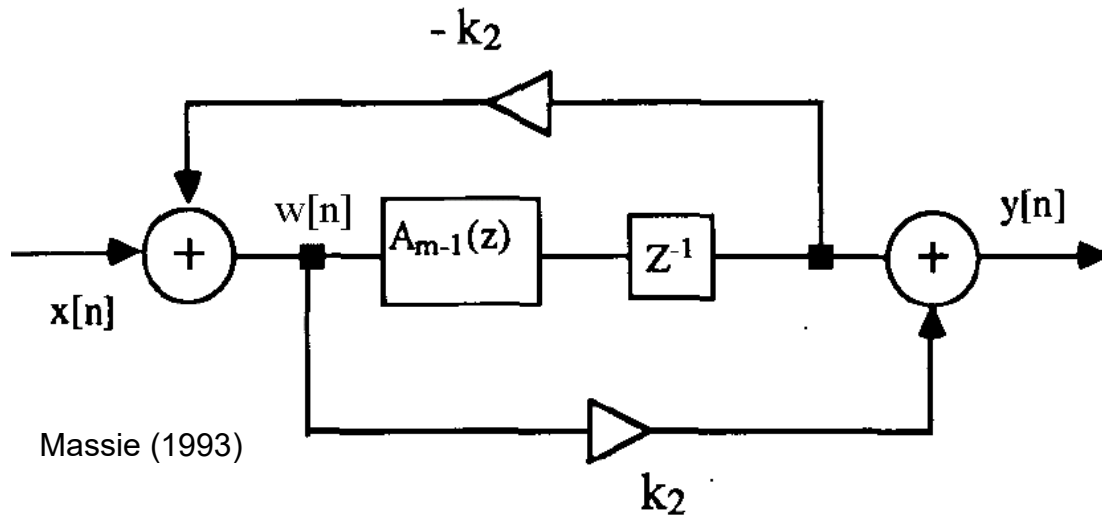


Datorro (1988)

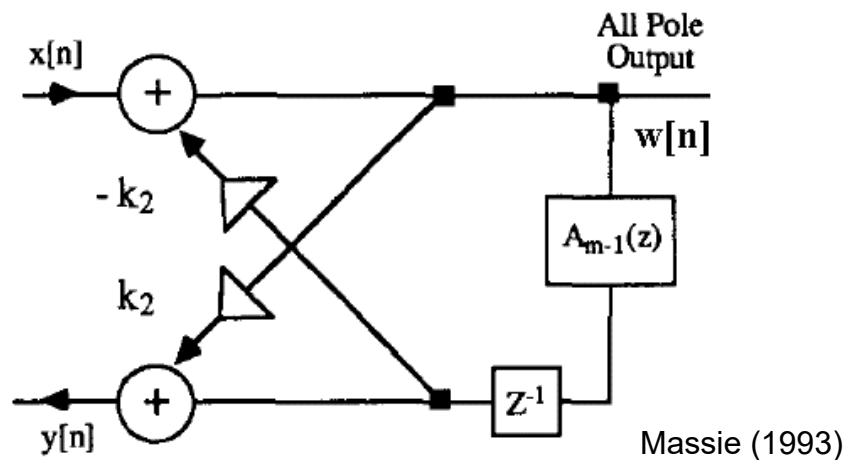


Massie (1993)

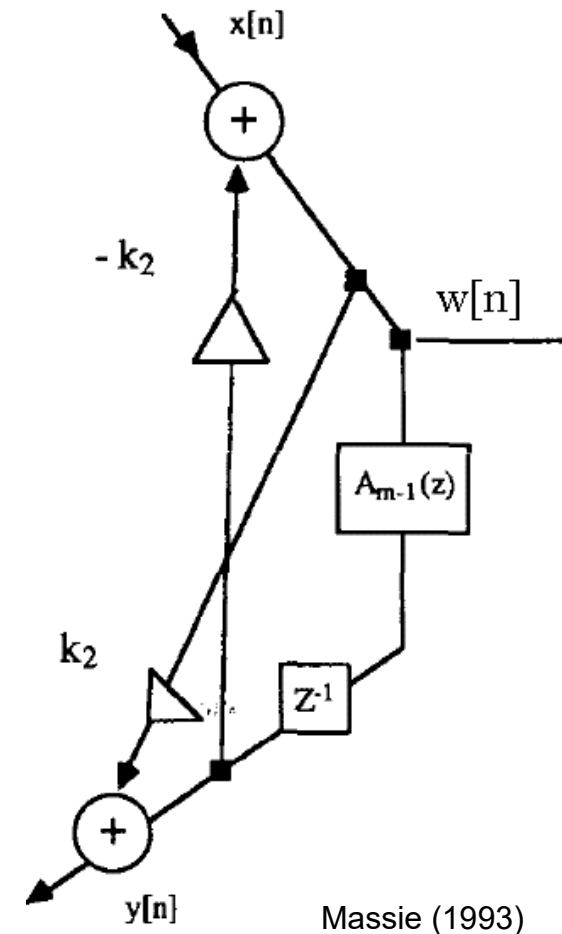
# Lattice Filter (2)



Massie (1993)



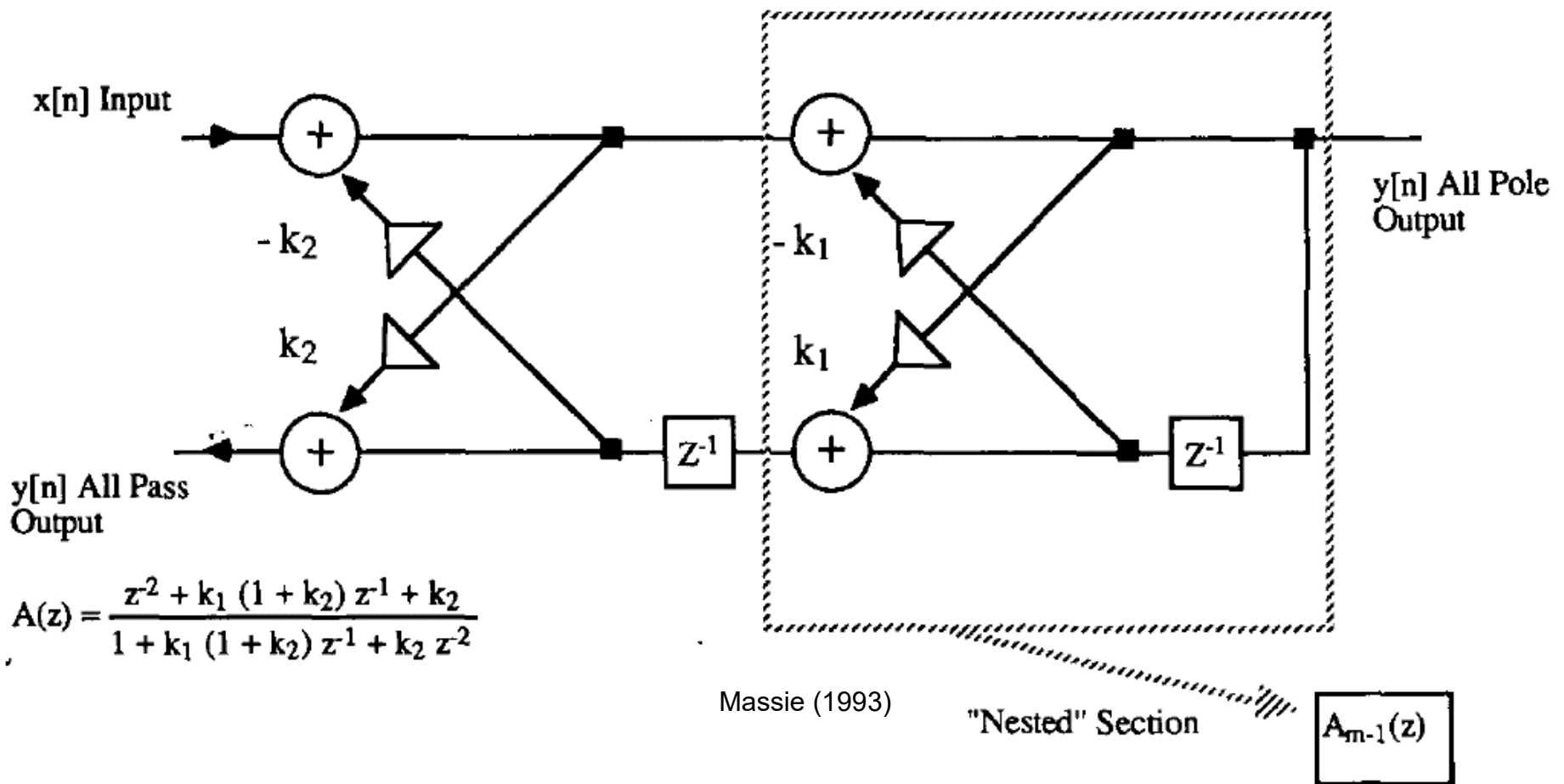
Massie (1993)



Massie (1993)

# Lattice (3)

- Ergebnis:



$$A(z) = \frac{z^2 + k_1(1 + k_2)z^1 + k_2}{1 + k_1(1 + k_2)z^{-1} + k_2z^{-2}}$$

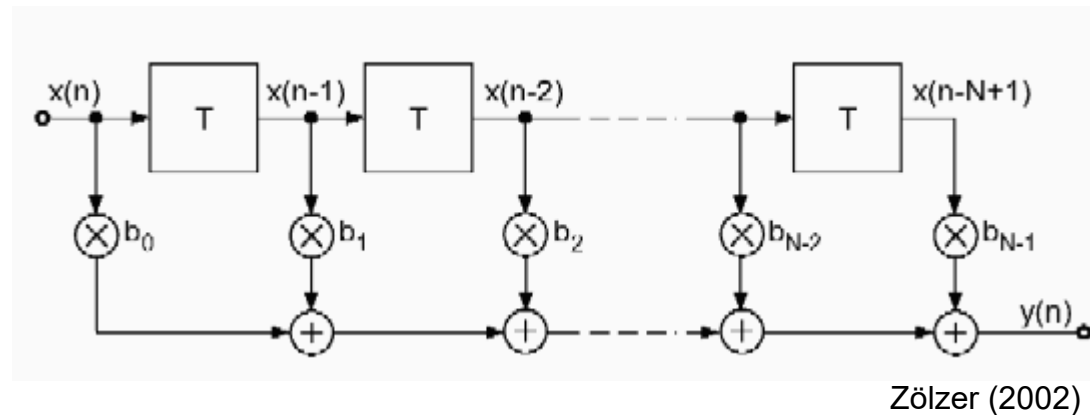
# Nichtrekursive Audio-Filter

- Direkte Form: Faltung im Zeitbereich
- Schnelle Faltung: Multiplikation im Frequenzbereich
- Hybride Methoden:
  - Zero-delay convolution
  - Low-latency convolution



# Faltung im Zeitbereich

- Faltung: 
$$y[n] = (x * h)[n] = \sum_m^M x[m] \cdot h[n - m]$$



- Einfache Realisierung
- Zeitintensive Berechnung:  $O(N \cdot M)$   
 ( $N$  ... Signallänge)  
 ( $M$  ... Filterlänge)

# Schnelle Faltung

- Multiplikation im Frequenzbereich:  $Y[k] = X[k] \cdot H[k]$

$x[n]$
--------

Länge: L

$h[n]$
--------

Länge: M

Zero padding auf Länge:  $N = L + M - 1$

$x[n]$	0
--------	---

 $\rightarrow DFT \rightarrow$ 

$X[k]$
--------

$h[n]$	0
--------	---

 $\rightarrow DFT \rightarrow$ 

$H[k]$
--------

Faltung durch Multiplikation:

$y[n]$
--------

 $\swarrow$  IDFT  $\swarrow$ 

$Y[k] = X[k] \cdot H[k]$
--------------------------

Länge:  $N = L + M - 1$ Aufwand:  $O\left(\frac{3}{2} \cdot N \cdot \log_2 N\right) + 3N$

# Schnelle Faltung – optimiert



- Bildung einer komplexwertigen Folge:

$$z[n] = x_F[n] + j \cdot x_{F+1}[n]$$

- Zero padding von  $h[n]$  und  $z[n]$  auf  $N = M + L - 1$
- DFT von  $h[n]$  und  $z[n]$ :  $H[k]$  und  $Z[k]$
- Multiplikation und IDFT:

$$e[n] = (z * h)[n] = (x_F * h)[n] + j(x_{F+1} * h)[n]$$

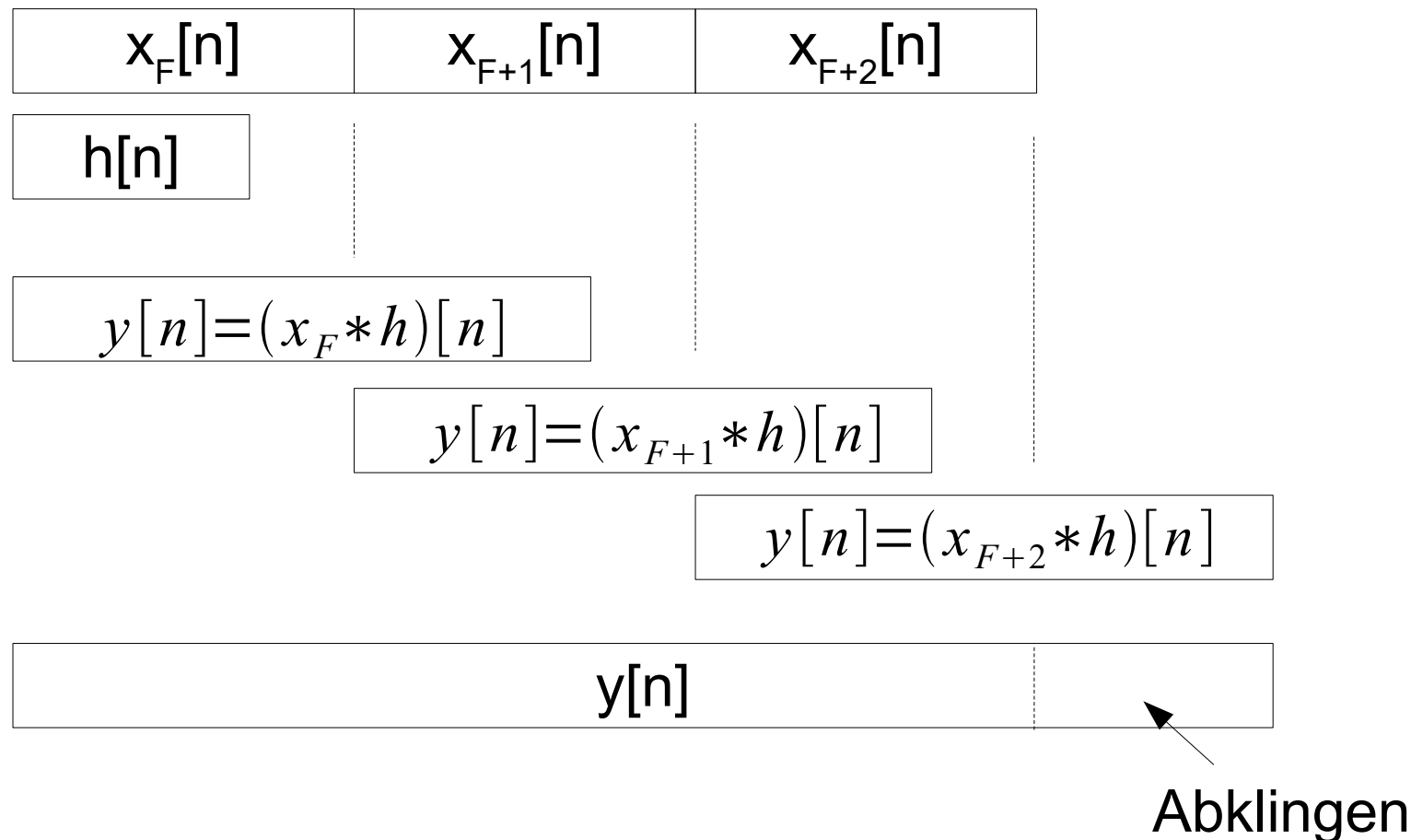
- Folgen teilen:

$$y_F[n] = \Re\{e[n]\}$$

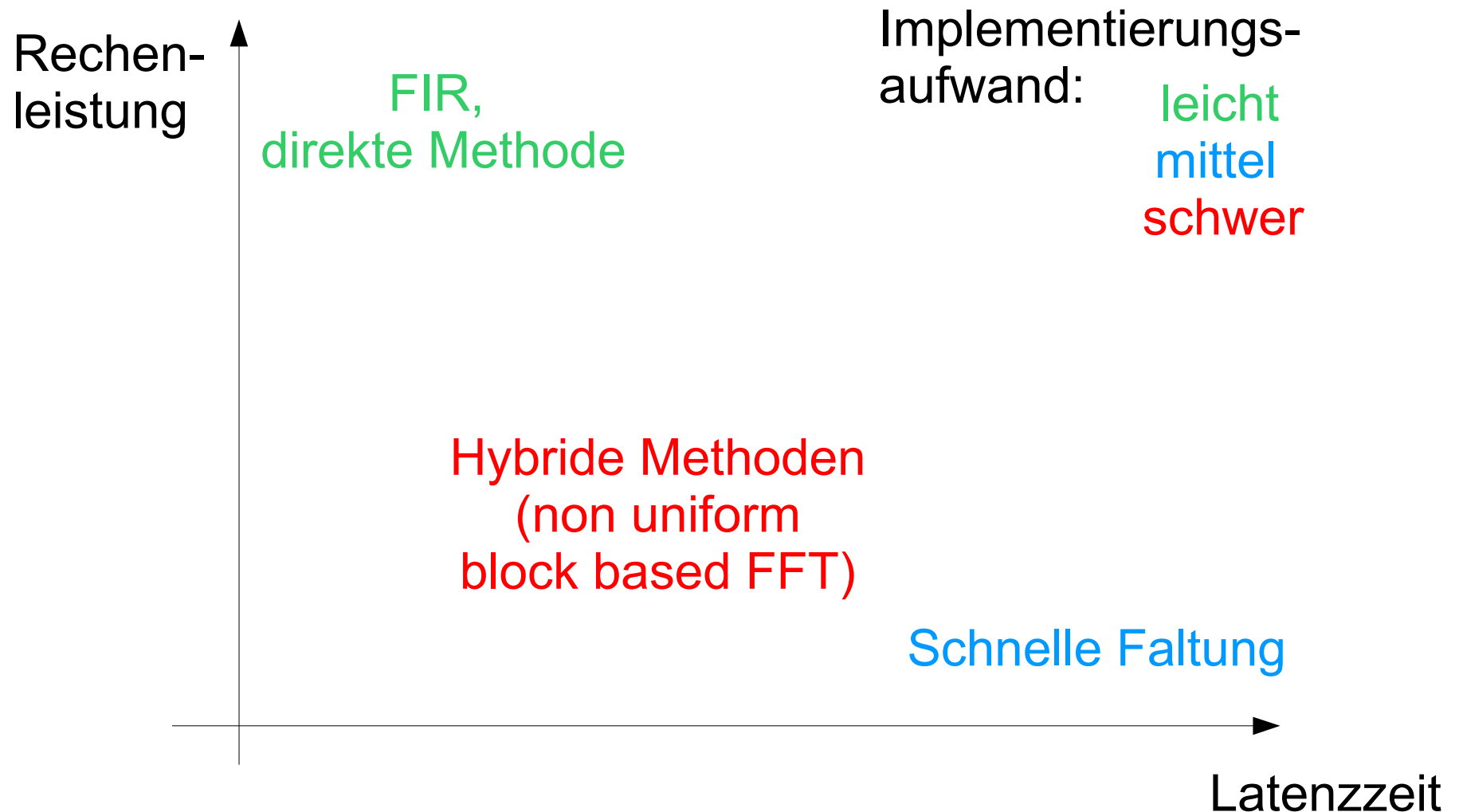
$$y_{F+1}[n] = \Im\{e[n]\}$$

# Zusammenführen der Ergebnisse

- Methode: Overlap and Add:



# Vergleich der Faltungsalgorithmen

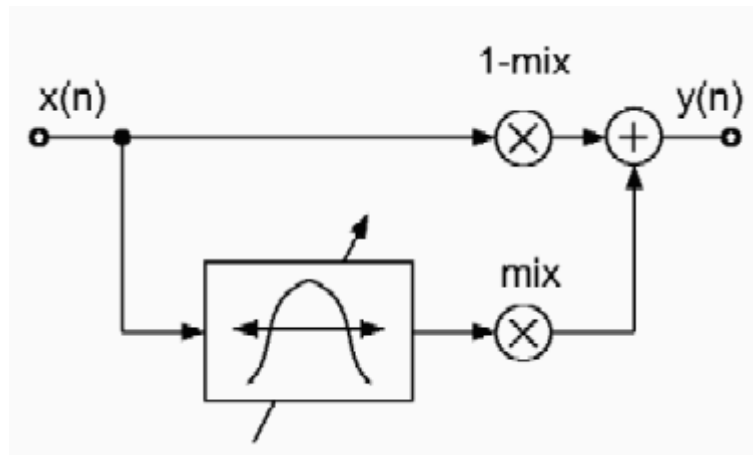


# Zeitvariante Filter

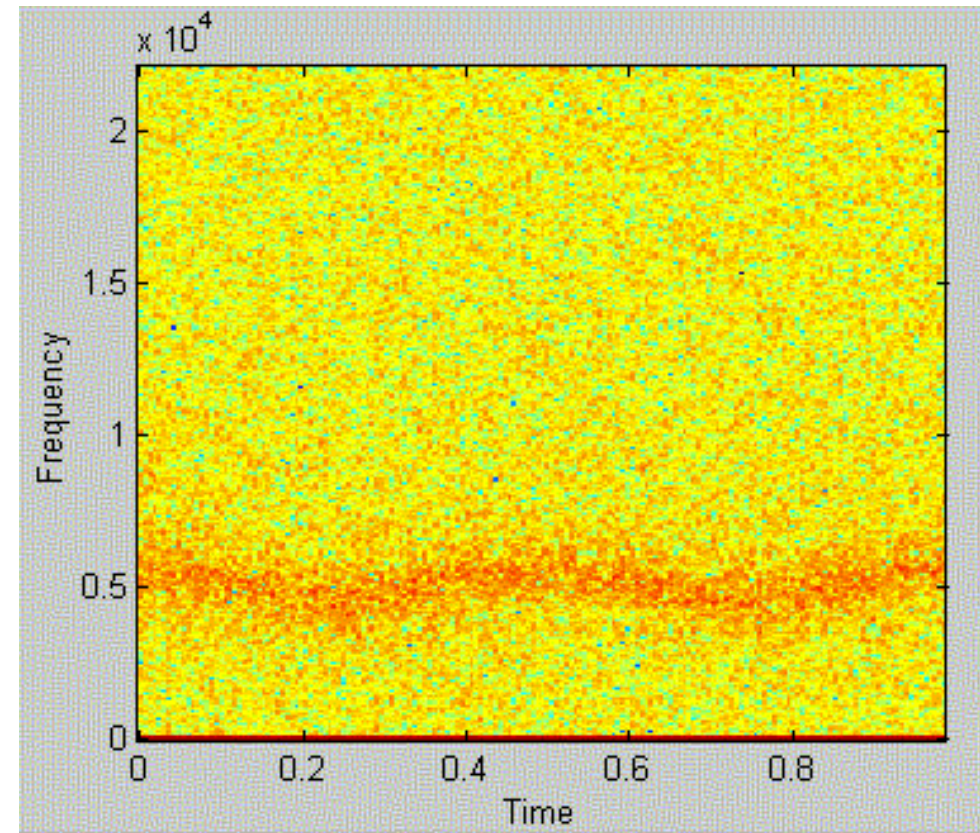
- Synthesizer
- Equalizer
- Echtzeitssysteme
- Virtuelle Akustik
- Beispiele:
  - Wah-wah
  - Phaser

# Wah-Wah

- Zeitvarianter Peak-Filter:

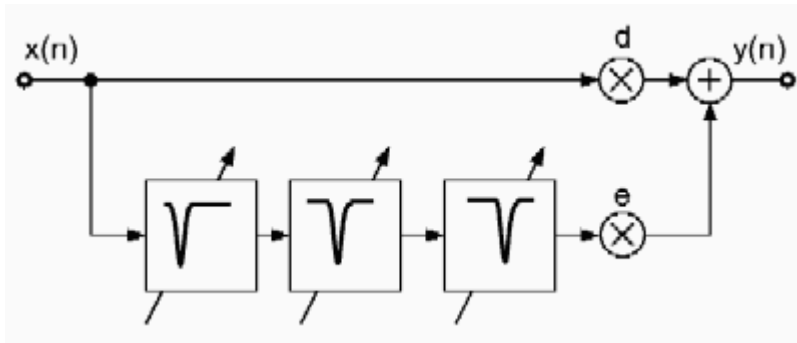


Zölzer (2002)

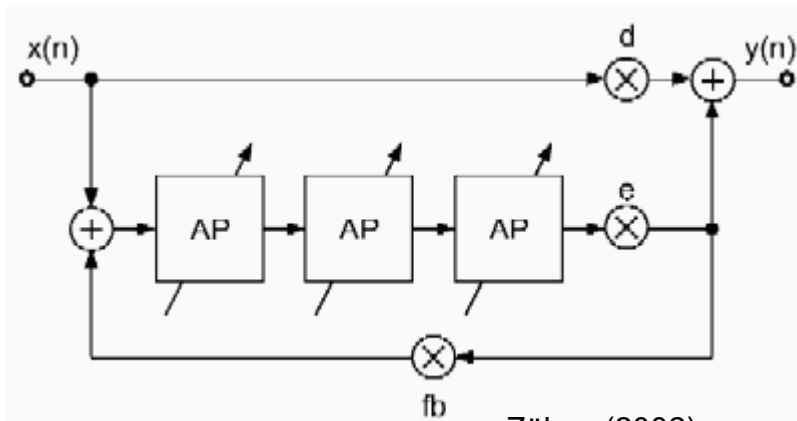


# Phaser

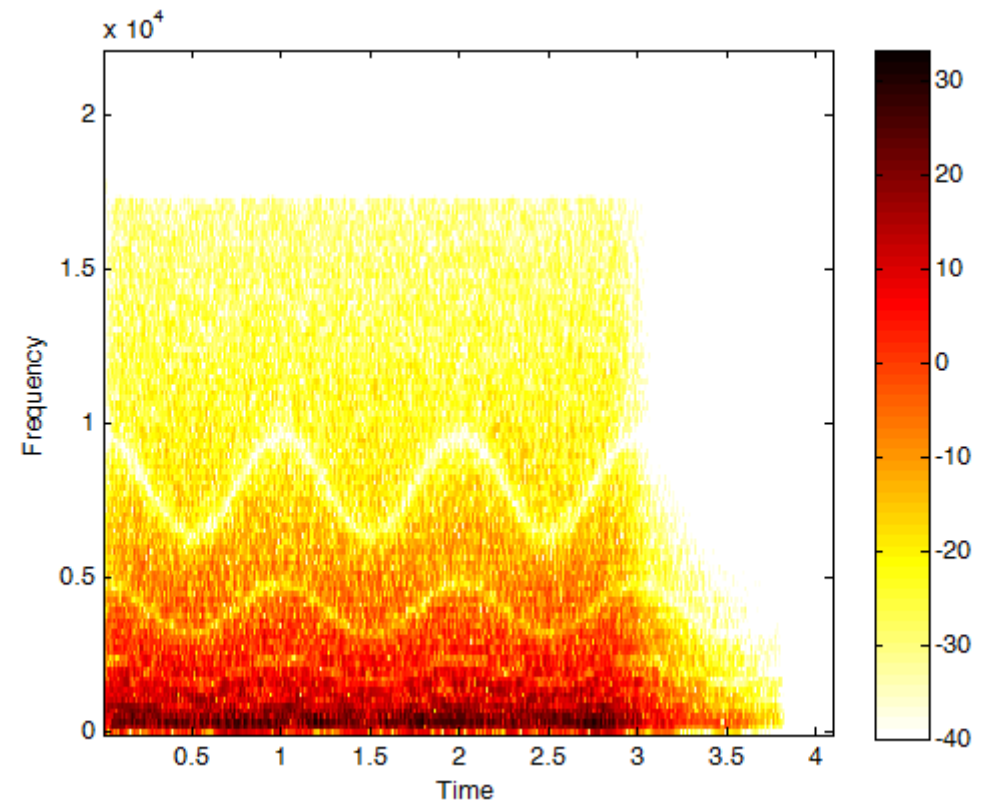
- Mehrere zeitvariante Notch-Filter:



Zölzer (2002)



Zölzer (2002)





# Weiterführende Literatur

- Jon Datorro (1988). „The Implementation of Recursive Digital Filters for High-Fidelity Audio,“ J. Audio Eng. Soc., 36: 851-878.
- Dana C. Massie (1993). „An Engineering Study of the Four-Multiply Normalized Ladder Filter,“ J. Audio Eng. Soc., 41: 564-582.
- Udo Zölzer (1994). „Roundoff Error Analysis of Digital Filters,“ J. Audio Eng. Soc., 42: 232-244.
- Rhonda Wilson (1993). „Filter Topologies,“ J. Audio Eng. Soc., 41: 667-677
- Gardner, W. G. (1995). "Efficient convolution without input-output delay," J Audio Eng Soc 43, 127-136.